

§ 7 蜗杆传动 worm (shaft angle交错轴 $\Sigma = 90^\circ$)

- ❖ § 7-1 General considerations 概述
- ❖ § 7-2 Choice of the parameters and geometrical calculations
Archimedean worm gearing 普通圆柱蜗杆传动参数选择和几何计算
- ❖ § 7-3 Materials, types of failure and design criteria
蜗杆传动材料、失效和计算准则
- ❖ § 7-4 Forces in worm gearing 受力分析
- ❖ § 7-5 Strength calculations 强度计算
- ❖ § 7-6 Heat balance calculation 热平衡计算
- ❖ Design example 习题课¹

物理与电气工程学院

§ 7-1 General considerations 概述

- ❖ Characteristics and applications

特点和应用

- ❖ Types of the worm gearing

分类

物理与电气工程学院

2
回总目录

一、Characteristics and applications 特点和应用

1、Advantages

① High speed ratio $i \rightarrow$ Small overall size

($Z=1\sim4$) 结构紧凑

{ For motion drives 传递运动: Up to 1000

{ For power drives 传递动力: 8~80

② Smooth and silent operation 平稳, 噪音小 (Continuous engagement between worm and worm

gear

蜗杆与蜗轮连续啮合)

3

2、Drawbacks

① Low efficiency 效率低

($\Sigma=90^\circ$, $V_s \uparrow$) Usually $P < 50\text{KW}$

② High cost 成本高

(The necessity of using expensive antifriction materials for the worm gear 蜗轮轮圈用贵重有色金属青铜制造)

3、 Applications应用

二、 Types 分类：

- ❖ 1、 Cylindrical worm gearing 圆柱蜗杆传动：
 - ❖ By shape of the threads in an end section
按轴截面内螺旋线形状分：
 - ① Archimedean worm 阿基米德蜗杆 (Ordinary 普通)
 - ② Involute worm 渐开线蜗杆
 - ③ Convolute worm 延伸渐开线蜗杆 (Straight-sided in the section normal to the thread 法向直廓)
- ❖ 2、 Enveloping worm gearing 环面蜗杆传动
- ❖ 3、 Cone worm gearing 锥蜗杆传动



三、Meshing features of the ordinary cylindrical worm gearing 普通圆柱蜗杆传动啮合特点

❖ Central cross section 中间平面（主平面）：

A plane which is normal to the worm gear shaft and parallel to worm shaft

通过蜗杆轴线并垂直于蜗轮轴线的平面

Special features 特点：

In the central cross section 中间平面内

The tooth profile on worm is straight-sided

蜗杆齿廓：直线

The tooth profile on worm gear is in

volute 蜗轮齿廓：渐开线

Mesh between a straight-sided rack and an involute gear

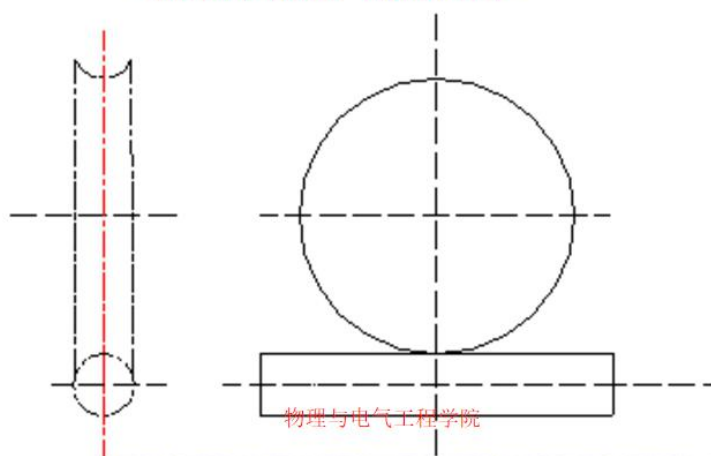
为（直）齿条与渐开线齿轮啮合



❖特点： 中间平面内

蜗杆齿廓：直线
蜗轮齿廓：渐开线

为（直）齿条与
渐开线齿轮啮合



9

BACK



§7-2 Choice of the parameters and geometrical calculations for Archimedean worm gearing

普通圆柱蜗杆传动参数选择和几何计算

- ❖Choice of the parameters 参数选择
- ❖Geometrical calculations 几何计算

物理与电气工程学院

10

回总目录



一、Choice of parameters 参数选择

1、Modules 模数 m ,

Profile angle 齿形角 α

In a central cross section,
modules and profile angle are standardized

中间平面内为标准值

$\alpha = 20^\circ$ m —— P84 表6-4

2、 d_1 , q

d_1 ——The pitch diameter of a worm
蜗杆中圆直径

q ——The worm diameter factor
蜗杆直径系数

① Gear 齿轮 $d_1 = mZ_1$

If m and Z_1 are determined, d_1 is determined.

② Worm 蜗杆 $d_1 = mZ_1 / \tan \gamma \neq$

mZ_1 if m and Z_1 are determined, d_1 is indeterminate.

③ Number of the hobs cutting worm gears depends on the diameter

❖ d_1 In order to limit number of the hobs and standardize them, several standard diameters are specified

❖ d_1 多, 则加工蜗轮滚刀多 为减少刀具数目, 并便于实现标准化、系列化:
对每一 m 蜗杆规定几个标准 d_1
P84—表6—4

$$\frac{d_1}{m} = q \quad \text{蜗杆直径系数} \quad d_1 = mq \neq mZ_1$$

3、 i 、 Z_1 、 Z_2



① Z_1 the number of worm 蜗杆头数

1, 2, 4, 6

P83: $Z_1 = f(i) \left\{ \begin{array}{l} \star Z_1 \downarrow \quad i \uparrow \quad \eta \downarrow \\ \star Z_1 \uparrow \quad \eta \uparrow \quad i \downarrow \end{array} \right. \text{加工难}$

② i Speed ratio 传动比

$$i = \frac{n_1}{n_2} = \frac{Z_2}{Z_1} \neq \frac{d_2}{d_1}$$



③ $Z_2 = i Z_1$

$28 \leq Z_2 \leq 100$ (32~63 is optimal for power drives 一般32~63)

★ $\left\{ \begin{array}{l} Z_2 < 28, \text{ Stationary 则平稳性} \downarrow \end{array} \right.$

★ $\left\{ \begin{array}{l} Z_2 > 100, \text{ 则 } d_2 = m Z_2 \uparrow \end{array} \right.$

Longer distance between the bearings of the worm
→ 杆长 \uparrow → Rigidity of worm 杆刚度 \downarrow

④ γ The pitch helix angle of the worm
蜗杆导程角

$$\operatorname{tg} \gamma = \frac{p_x Z_1}{\pi d_1} = \frac{\pi m Z_1}{\pi m q} = \frac{Z_1}{q}$$

$\gamma = 3.5^\circ \sim 33^\circ$ $\left\{ \begin{array}{l} \gamma > 33^\circ \quad \eta \text{ Increases slightly} \\ \text{Difficulties in} \\ \text{manufacture} \end{array} \right.$

\uparrow 缓慢, 加工难 locking 自锁 η 低



• 二、 Geometrical calculations

• Application 传动功用、 $i \rightarrow Z_1$ ($Z_2 = i Z_1$)

• Strength conditions 强度条件, q

$$\gamma = \tan^{-1} \frac{Z_1}{q}$$



P78 表4—5 Geometrical dimensions are calculated

计算几何尺寸.

物理与电气工程学院

17

BAC



§ 7—3 Materials, types of failure and design criteria
蜗杆传动材料、失效和计算准则

- ❖ Materials 材料
- ❖ Types of failure 失效形式
- ❖ Design criteria 计算准则
- ❖ 则 Structure and manufacture of the worm and worm gear
蜗轮蜗杆的结构及加工方法

物理与电气工程学院

18

回总目录

十、Materials 材料



Requirements 要求:

Not only have enough strength And the good antifriction is more important

不仅要有一定强度，更重要的是要有良好的减摩性和耐磨性。

(Material pairs of low and high levels of hardness 软硬相配)

1、Worm materials 蜗杆

Usually : 45

Structurally improvement 调质

High velocities and heavy load 高速重载:

45、40Cr、20Cr

Hardening 淬火





2、Worm gear materials 蜗轮

- ❖ High velocities and heavy load 高速重载
($V_s = 12 \sim 25 \text{m/s}$)

Tin bronze 锡青铜

{ ZCuSn10P1
ZCuSn5Pn5Zn5

Better seize-resistant lower strength higher cost

抗胶合能力强，机械强度低，价格高

($\sigma_b < 300 \text{MPa}$)

物理与电气工程学院

21

Usually : Aluminum-Iron bronze 铸铝青铜 ($V_s = 2 \sim 8 \text{m/s}$)

- ❖ ZCuAl10Fe3Mn2
- ❖ ZCuAl8Mn13Fe3Ni2

Lower seize-resistant ,Higher strength
Lower cost

抗胶合能力差，机械强度高，价格低

($\sigma_b > 300 \text{MPa}$)

物理与电气工程学院

22



❖ Low velocities and light load 低速轻载:

$$(V_s < 2\text{m/s})$$

Cast iron 灰铸铁

{ HT200
HT150

物理与电气工程学院

BACK

二、Types of failure 失效形式

❖ Similar to types of failure in the toothed gearing
Fatigue pitting, Seizing, Abrasive wear, Plastic
deformation and tooth breakage

与齿轮类似：点蚀、胶合、磨损、塑性
变形、断齿

❖ Special features as below 特点如下：

① Worm gear failure 蜗轮失效

(Strength of the worm is more than the one of worm

gear 蜗杆强度高于蜗轮)

② Closed drives 闭式传动:

Tin bronze 锡青铜:

The principal types of failure are pitting (点蚀为主) because it has good seize-resistant ability (抗胶合能力强)

Aluminum-iron bronze and cast iron 无锡青铜和铸铁:

The principal types of failure are seize (胶合为主)



③ Open drive 开式传动:

Breakage after severe wear 磨损→折断

三、Design criteria 计算准则



Closed drive 闭式传动

- ❖ ① Designed according to contact fatigue strength of the tooth surface

按齿面接触疲劳强度设计

- ❖ ② Checked according bending fatigue strength of the tooth root 按齿根弯曲疲劳强度校核

- ❖ ③ Heat balance calculation 热平衡计算

Open drive 开式传动:

Design according to bending fatigue strength

按弯曲疲劳强度设计

物理与电气工程学院

27

四、Structure and manufacture of the worm and worm gear



蜗轮蜗杆的结构及加工方法

1、Structure 结构

- ① Worm gear 蜗轮

- ② Worm 蜗杆

2、Manufacture 加工方法

- ① Worm gear 蜗轮

- ② Worm 蜗杆

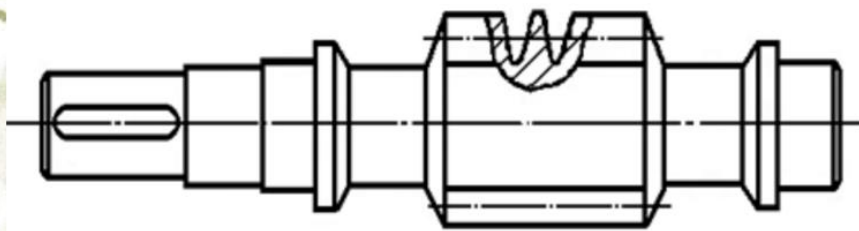
物理与电气工程学院

28

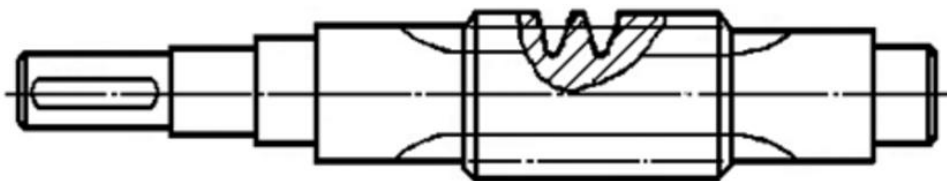


Worms are made integral with the shaft and are seldom made fabricated, the following types of the worm designs are used

蜗杆通常与轴做成整体，很少做成装配式，可分为：



With tool escape 有退刀槽



Without tool escape 无退刀槽

§ 7-4 Forces in worm gearing



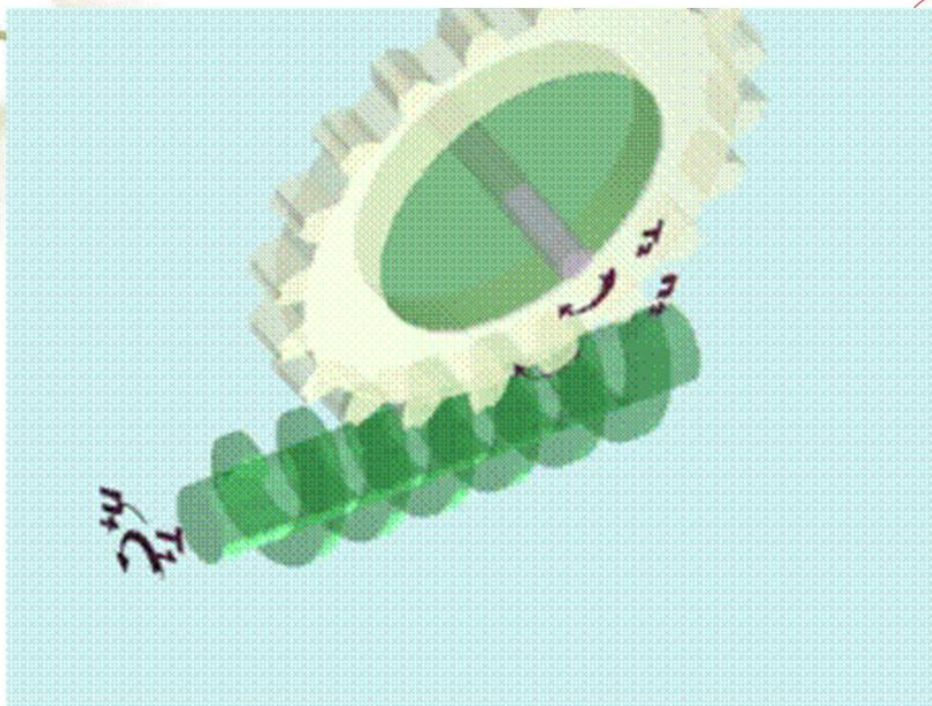
受力分析

- ❖ Relationship between forces acting on worm and worm gear 各力间关系
- ❖ Magnitude of forces 力的大小
- ❖ Direction of forces 力的方向

物理与电气工程学院

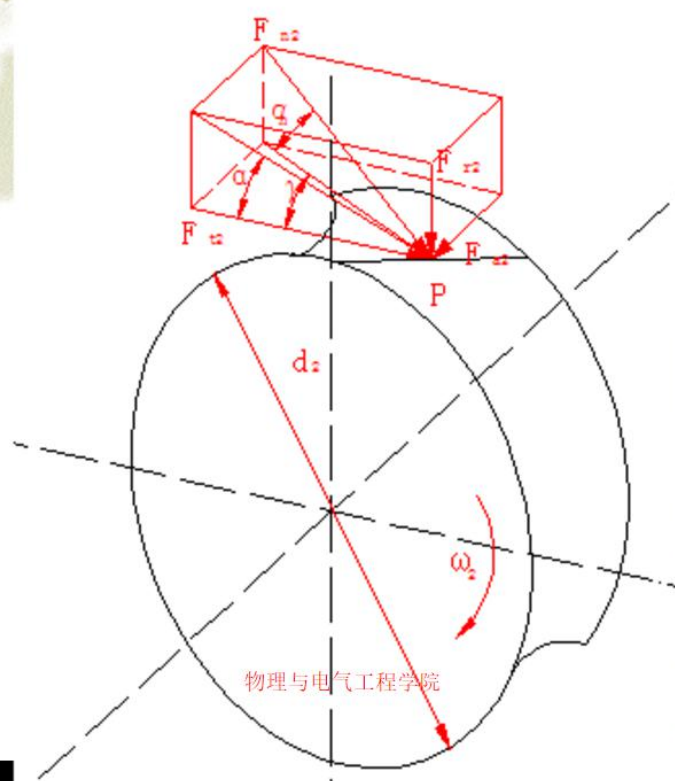
31

回总目录

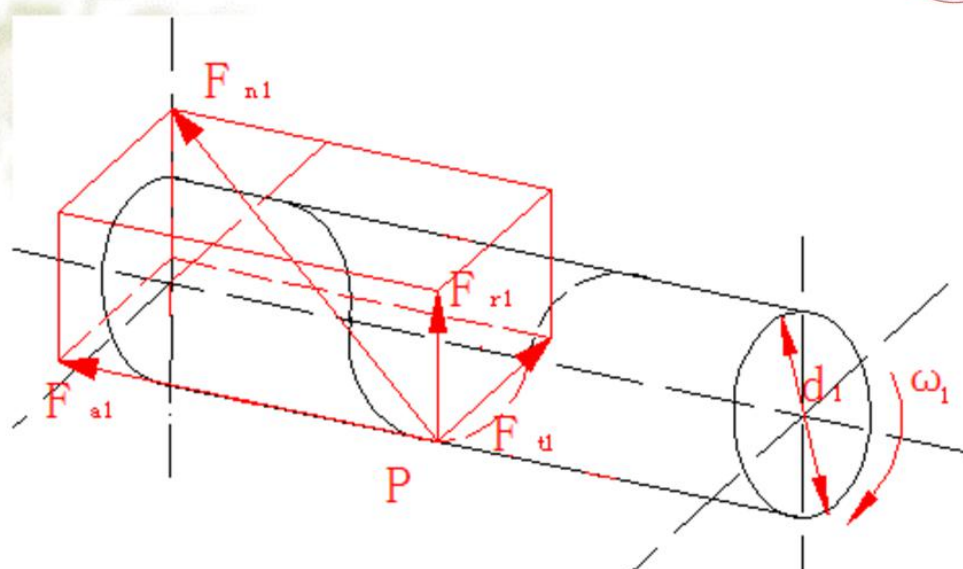


物理与电气工程学院

32



33



物理与电气工程学院

34

一、Relationship between forces 各力间关系



$$\begin{cases} \vec{F}_{r1} = -\vec{F}_{r2} \\ \vec{F}_{t1} = -\vec{F}_{a2} \\ \vec{F}_{a1} = -\vec{F}_{t2} \end{cases}$$

物理与电气工程学院

35

[BACK](#)

二、Magnitude of forces 力的大小



$$\begin{aligned} F_{t1} &= F_{a2} = \frac{2T_1}{d_1} \\ F_{a1} &= F_{t2} = \frac{2T_2}{d_2} \quad (T_2 = T_1 \bar{m}_1) \\ F_{r1} &= F_{r2} = F_{a2} \tan \alpha \\ F_{a1} &= F_{a1} = F_a = \frac{F_{a2}}{\cos \gamma \cos \alpha_a} = \frac{2T_2}{d_2 \cos \gamma \cos \alpha} \end{aligned}$$

(γ_{\max} 处, $\cos \alpha_a$ 和 $\cos \alpha$ 相差 0.01)

物理与电气工程学院

36

[BACK](#)

三、Direction of forces 力的方向



① F_t 和 F_r 同前 As before

② F_a (蜗杆主动):
Worm driving

{	①旋向: 左旋伸左手, 右旋伸右手
	②回转方向: 四指握拳方向
	③ F_a 方向: 与拇指指向相同

(F_2 与 F_a 相反, ω_2 与 F_2 相同)

Helix direction: left hand for left-hand helix gear
right hand for right-hand helix gear.

3 Rotating direction: bending direction of
fingers.

Direction of F_{a1} : The same direction of thumbing

物理与电气工程学院

37

BACK

§ 7—5 Strength calculation



强度计算

❖ Contact fatigue strength of the
tooth surface on worm gear

蜗轮齿面接触疲劳强度计算

❖ Bending fatigue strength of the
tooth root on worm gear

蜗轮齿根弯曲疲劳强度计算

物理与电气工程学院

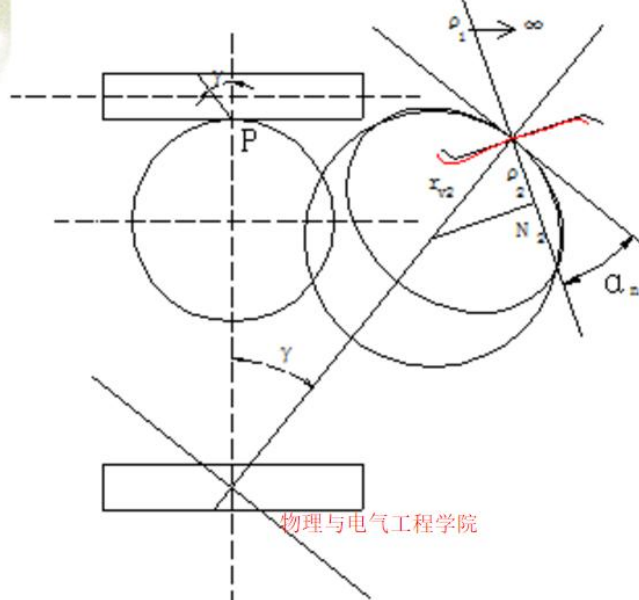
38

回总目录

一、蜗轮齿面接触疲劳强度计算



Contact fatigue strength of the tooth surface on worm gear



39

In the central cross section, the profiles of the worm and worm gear are the same as that of involutes helical gear and rack. Hence, the strength calculations are done for the tooth profiles of their equivalent spur gear and rack .

在主平面内，蜗轮蜗杆的啮合相当于一对渐开线斜齿轮与斜齿条的啮合→取其当量直齿轮与直齿条的啮合进行强度计算。

物理与电气工程学院

40



$$\sigma_H = \sqrt{\frac{F}{\pi L} \frac{\frac{1}{\rho}}{\left(\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}\right)}} \leq [\sigma]_H \dots \textcircled{1}$$

$$\textcircled{1} \quad \frac{1}{\rho} = ?$$

$$\gamma_{v2} = \frac{d_2}{2\cos^2\gamma}$$

$$\rho_2 = \gamma_{v2} \sin\alpha_n = \frac{d_2 \sin\alpha_n}{2\cos^2\gamma}$$

$$\frac{1}{\rho} = \frac{\rho_1 + \rho_2}{\rho_1 \rho_2} = \frac{1 + \frac{\rho_2}{\rho_1}}{\rho_2} = \frac{1}{\rho_2} = \frac{2\cos^2\gamma}{d_2 \sin\alpha_n} \dots \textcircled{2}$$

41

❖ ②



$$F = K F_n = \frac{2KT_2}{d_2} \frac{1}{\cos\alpha \cos\gamma} \dots \textcircled{3}$$

❖ ③ Length of the contact line 接触线长度 L

$$\widehat{AB} = \frac{\theta}{360^\circ} \pi d_1$$

物理与电气工程学院

42

计入:

a、多对齿啮合 $\rightarrow \times \varepsilon_a$

b、齿倾斜 $\rightarrow \div \cos \gamma$

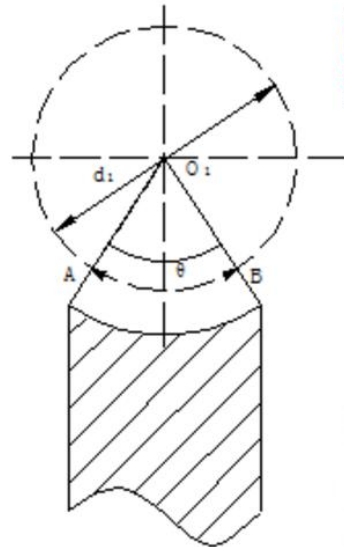
c、接触线长度变化系数 $\rightarrow \chi$

$$L_o = AB \frac{\varepsilon_a \chi}{\cos \gamma} = \frac{\pi d_1 \chi \varepsilon_a \theta^\circ}{360^\circ \cos \gamma}$$

$$\varepsilon_a = 2$$

$$\theta = 100^\circ$$

$$\chi_{\min} = 0.75$$



❖ Taking into account the following factors:

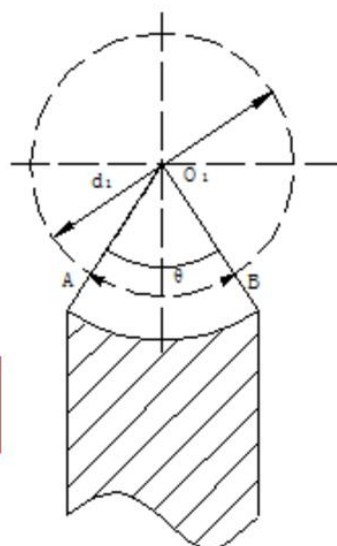
a、 More than one pair of teeth mesh each other 多对齿啮合 $\rightarrow \times \varepsilon_a$

b、 The teeth are inclined to the elements of the cylinders of the worm 齿倾斜 $\rightarrow \div \cos \gamma$

c、Change factor of the contact line length 接触线长度变化

系数 $\rightarrow \chi$

$$L_n = AB \frac{\varepsilon_a \chi}{\cos \gamma} = \frac{\pi d_1}{360^\circ} \frac{\chi \varepsilon_a \theta^\circ}{\cos \gamma}$$



$$\varepsilon_a = 2$$

$$\theta = 100^\circ$$

$$\chi_{\min} = 0.75$$

④ Substituting equations 把式②、③、④ 代入 into equation①:



$$\sigma_H = \sqrt{\frac{1}{\pi \left(\frac{1 - \mu_1^2}{E_1} + \frac{1 - \mu_2^2}{E_2} \right)} \frac{2KT_2}{d_2} \frac{1}{\cos \alpha \cos \gamma} \frac{2 \cos \gamma}{d_2 \sin \alpha}} = Z_E \sqrt{\frac{9.47 KT_2}{d_1 d_2^2} \cos \gamma} \leq [\sigma]_H$$

$$\sigma_H = Z_E \sqrt{\frac{9.47 KT_2}{d_1 d_2^2} \cos \gamma} \leq [\sigma]_H \quad \text{For checking (校核公式)}$$

⑤ Substituting $d_1=mq$ and $d_2=mz_2$ into above equation :

$$m^3 q \geq 9.47 \cos \gamma \cdot K T_2 \left(\frac{Z_E}{Z_2 [\sigma]_H} \right)^2$$

For design
(设计公式)

2、 Notes:



① $[\sigma]_H$:

★ Tin bronze 锡青铜 ($\sigma_b \leq 300\text{MPa}$) :

The principal type of failure is pitting depend on the number of stress cycles
点蚀为主, $[\sigma]_H$ 与应力循环次数 N 有关;

$$[\sigma]_H = (0.79 \sim 0.9) \sigma_b \sqrt{\frac{10^7}{N}} \quad (N = 60 n_z t_h)$$

★ Aluminium-iron bronze 无锡青铜

($\sigma_b > 300\text{MPa}$) :

Principal type of failure is seizure,
independent of the number of stress cycles, but
depends on sliding velocity.

胶合为主, $[\sigma]_H$ 与应力循环次数 N 无关而与相对滑
动速度 V_s 有关。

$$[\sigma]_H = 300 - 25V_s \quad (\text{铜})$$

$$[\sigma]_H = 210 - 35V_s \quad (\text{铁})$$



$$\textcircled{2} K = K_A K_V K_\beta$$

Steady load $\rightarrow K_A, K_V, K_\beta$

Smaller

values of K_A, K_V, K_β 见P80 表4-6及P80说明

二、Bending fatigue strength of tooth root of worm gear 蜗轮齿根弯曲疲劳强度计算

By analogy with helical gearing, but stress concentration factor Y_{sa} not considered

按斜齿轮方法计算但不计应力修正系数 Y_{sa}

★①

$$\begin{cases} m_n = m \cos \gamma \\ b = \pi d_1 \theta^\circ / 360^\circ \cos \gamma \\ Y_\epsilon = 1 / \chi_{\epsilon a} \\ F_t = 2T_2 / d_2 \end{cases}$$

$$\sigma_F = \frac{KF_t}{bm_n} Y_F Y_\epsilon Y_\beta \leq [\sigma]_F$$

物理与电气工程学院

51

★② The teeth of wormgears are taken from 20 to 40% stronger than those of helical gears

弯曲强度与斜齿轮相比增大大约40% →

Strength increasing factor 强度增长系数 1.4

Tooth thickness grows

(Their arched tooth

shape results in so-called natural correction which occurs in all cross sections except the central one)

齿厚增大 (相当于正变位)



物理与电气工程学院

52

★③ The maximum allowable wear amount of the tooth root thickness is 20%

允许齿根厚度最大磨损20%

→ 补偿轮齿磨损系数1.5

$$\sigma_F = \frac{2KT_2}{d_2 m \cos \gamma} \frac{360^\circ \cos \gamma}{\pi d_1 \theta} Y_F Y_\beta \frac{1}{\chi \varepsilon_a} \frac{1.5}{1.4} \leq [\sigma]_F$$



$$\sigma_F = \frac{2KT_2}{d_2 m \cos \gamma} \frac{360^\circ \cos \gamma}{\pi d_1 \theta} Y_F Y_\beta \frac{1}{\chi \varepsilon_a} \frac{1.5}{1.4} \leq [\sigma]_F$$

取 $\theta = 100^\circ$ $\chi_{\min} = 0.75$ $\varepsilon_a = 2$

$$\sigma_F = \frac{1.64KT_2}{d_1 d_2 m} Y_F Y_\beta \leq [\sigma]_F \quad \text{(校核公式)}$$

For checking

$d_1 = mq$ $d_2 = mZ_2$ 代入

$$m^3 q \geq \frac{1.64KT_2}{Z_2 [\sigma]_F} Y_F Y_\beta \quad \text{(设计公式)}$$

For design



Notes说明

- ① Y_F , Form factor for worm gear teeth, selected according to the virtual number of teeth $Z_v = Z_2 / \cos^3 \gamma$.

Y_F 齿形系数, 根据 $Z_v = Z_2 / \cos^3 \gamma$ 查取 P82 表4-8

- ② Y_β Helix angle factor 螺旋角系数

$$Y_\beta = 1 - \gamma^\circ / 140^\circ$$

55

BAG

§ 4-6 Heat balance calculation



热平衡计算

❖ Efficiency 效率 $\eta = \eta_1 \eta_2 \eta_3$

❖ Heat balance calculation 热平衡计算



❖ 一、Efficiency 效率 $\eta = \eta_1 \eta_2 \eta_3$

❖ 1、 η_1 ——Efficiency in engagement

啮合效率 (The sliding velocity between tooth surfaces 齿面间相对滑动)

→ Power losses 功率损失

$$\eta_1 = \frac{\operatorname{tg} \gamma}{\operatorname{tg}(\gamma + \rho_v)}$$

❖ Mesh loss power is principal loss power. It is computed approximately by efficiency formula for screws

最主要功率损失近似用螺旋面效率计算公式

物理与电气工程学院

57

① Equivalent friction angle

ρ_v 当量摩擦角 = f (Pair of materials 材料组合、 Types of lubrication oils 润滑剂、 V_s)

$V_s \uparrow \rightarrow \rho_v \downarrow \rightarrow \eta \uparrow$

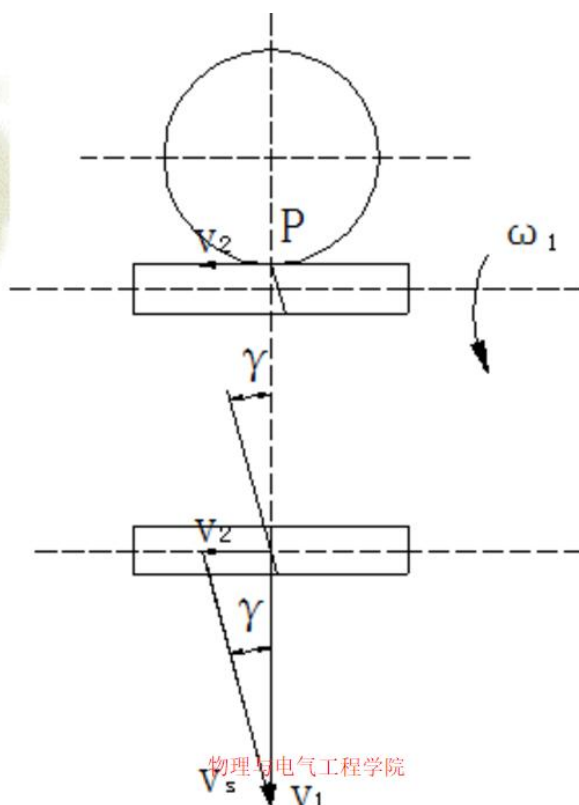
② The sliding velocity

V_s 相对滑动速度

$$\vec{V}_s = \vec{V}_1 - \vec{V}_2 \begin{cases} \text{方向: 螺旋线切线方向} \\ \text{大小: } V_s = \sqrt{V_1^2 + V_2^2} = \frac{V_1}{\cos \gamma} = \frac{V_2}{\sin \gamma} \end{cases}$$

物理与电气工程学院

58



物理与电气工程学院

59

- ③ The pitch helix angle of the worm threads
 γ 导程角



$$\gamma = \tan^{-1} \frac{Z_1}{q} \begin{cases} Z_1 \uparrow q \downarrow \rightarrow \gamma \uparrow \rightarrow \eta_1 \uparrow \\ Z_1 \downarrow q \uparrow \rightarrow \gamma \downarrow \rightarrow \eta_1 \downarrow \end{cases} \quad \begin{matrix} q \propto \text{杆刚度} \\ \text{Rigidity of the worm} \end{matrix}$$

2、 η_2 ——Efficiency in splashing about and mixing the lubricant 搅油效率 (≈ 0.99)

3、 η_3 ——Efficiency in bearing friction

轴承效率 (Rolling bearing 滚动轴承: 0.99

Sliding bearing 滑动轴承: 0.98~0.99)

物理与电气工程学院

60

BACK

二、Heat balance calculation



热平衡计算 ($t \leq 75^{\circ}\text{C}$ ~ 90°C)

The heat balance 热平衡 :

In an unit time, the amount of heat evolved by worm gearing equals to the heat removed by radiation from the free surface of housing and by conduction through the foundation plate or frame

传动发热速率 = 箱体散热速率

$$1000(1-\eta) P_1 = K_d A (t_1 - t_0)$$

$$\Delta t = t_1 - t_0 = \frac{1000(1-\eta) P_1}{K_d A}$$

$$t_1 = t_0 + \frac{1000(1-\eta) P_1}{K_d A} \leq 75^{\circ}\text{C} \sim 90^{\circ}\text{C}$$



★ Usually $t_0 = 20^\circ\text{C}$ (Temperature of the surrounding air)

环境温度 K_d ——Heat transfer factor 传热系数 $12 \sim 18 \text{ W/m}^2 \cdot ^\circ\text{C}$ (Larger values are used under favorable conditions of air circulation 通风良好取大值)

A ——Free surface of the housing from which heat is removed (m^2 , 散热面积) 外壁与空气接触, 内壁为油飞溅到或浸到的箱体表面积

As a first approximation 初算时:

Fins are located

$$A = 0.33 \left(\frac{a}{100} \right)^{1.75} \quad (\text{有散热片})$$

物理与电气工程学院

63



P_1 ——Power transmitted by worm

蜗杆传动功率 (KW)

★ If 若 $t_1 > 75^\circ\text{C} \sim 90^\circ\text{C}$, Then

① $\uparrow A \rightarrow$ Fins are located 散热片

② $\uparrow K_d \rightarrow$ Fan are mounted 风扇

$$K_d = 20 \sim 28 \text{ W/m}^2 \cdot ^\circ\text{C}$$

③ Water cooling is employed 循环水管

物理与电气工程学院

64

BACK