第六章 Toothed gearing 齿轮传动

§6-1 General considerations 概述

- Characteristics and applications 特点和应用
- Types of the toothed gearing 齿轮分类

一、Characteristics and applications 特点和应用

1、 Advantages 优点

① Long service life and high reliability 工作可靠，寿命长

②Constant speed ratio 传动比恒定

③ High efficiency 工作效率高

④Small overall size 结构紧凑

2、 Shortcomings 缺点

① High cost 成本高

② Noise in operation at considerable speeds or low precision in manufacture

制造精度低或者高速运转时，振动噪音大

③ They are unsuitable for duty over medium and long distances

不适合大的中心距

3、Applications 应用

同轴式双级圆柱齿轮减速器

二、 Types of toothed gearing 齿轮分类

1、 According to the difference between the relative positions of the shafts:

按轴布置方式分

① Toothed gearing between parallel shafts 平行轴齿轮传动

（ Cylindrical gearing 圆柱齿轮传动 ）

- Spur Cylindrical gearing 直齿圆柱齿轮传动
- Helical Cylindrical gearing 斜齿圆柱齿轮传动
- Herringbone gearing 人字齿圆柱齿轮传动

② Toothed gearing between intersecting shafts 相交轴齿轮传动（ Bevel gearing 圆锥齿轮传动）

Straight bevel gearing 直齿圆锥齿轮传动

Skew bevel gearing 斜齿圆锥齿轮传动

Spiral bevel gearing 曲齿圆锥齿轮传动

③Toothed gearing between crossed shafts ： 交错轴齿轮传动

Crossed helical gearing 螺旋齿轮传动

2、 According to the difference operating conditions: 按工作条件分

① Closed gearing 闭式齿轮传动

② Open gearing 开式齿轮传动

③ Partly open gearing 半开式齿轮传动

§ 3-2 Materials, types of failures and design criteria of gears

齿轮材料、失效形式和计算准则

- Materials of gears 齿轮材料
- Types of failures 失效形式
- Design criteria 计算准则

一、 Materials of gear 齿轮材料

Requirements 基本要求：

① Enough impact toughness

一定的冲击韧性

(Ensure sufficient beam strength in bending under variable load)

（保证变载荷下，弯曲疲劳强度较高)

② Enough hardness 一定的硬度

（ Ensure sufficient endurance of the surface layers

保证抗齿面失效的能力）

③High manufacturability and low cost 工艺性好、成本低

1、 Forged steel 锻钢

（ Wide application 常用）

(1) Under ordinary service condition一般场合：Lower hardness of tooth surface 软齿面 HB≤350）齿轮材料

Processing procedure 工艺过程：

Medium-carbon steels(45) 中碳钢 、Medium-carbon alloy steels(40Cr) 中碳合金钢

Structural improvement and normalization ——调质或正火

Cutting 切齿

For gears with lower hardness of tooth surface, the hardness difference between small gear tooth surface and large gear one is required:

软齿面大小齿轮之间要求具有一定的硬度差:

$\Delta HB = HB_1 - HB_2 = 30 \sim 50$

(2) Critical gears 重要应用:

Higher hardness of tooth surface 选硬齿面HB>350）齿轮材料

❖ Processing procedure 工艺过程:

Cutting teeth 切齿

Hardening 淬火

Grinding 磨削

❖ 2、Cast steel 铸钢

（For gears of large diameters

尺寸较大、不容易锻造时选用 $d_a \geq 400mm$）

3、 Cast iron铸铁

(1) Grey iron灰口铸铁（HT200~HT400)：

Easy to manufacture and low cost

工艺性好、价格低；

However, the strength and toughness is considerably less than that of steel

冲击韧性差、强度低

For low-speed, large-size gearing without impact:

低速、无冲击、大尺寸齿轮

(2) High-strength cast iron 球墨铸铁（QT500－5、 QT600－2等)：

High strength and toughness

Taking the place of cast steel gears

强度高、冲击韧性好、代替铸钢

4、 Plastics非金属材料（ Caprolon and polyformaldehyde etc 夹布胶木和尼龙等, ）:

Self-lubrication and noiseless operation具有自润滑性、工作噪音低

For gears of high speed, light load and low accuracy requirement

用于高速、小功率、精度要求不高时

二、Types of failure 失效形式

Tooth breakage轮齿折断

Failure of tooth surface layers 齿面损伤: Fatigue pitting, seizing, abrasive wear and plastic flow of material 点蚀、胶合、磨损和塑性变形

三、Design criteria 计算准则

- 1、 Close gearing闭式齿轮传动： The principal types of failure are pitting, tooth breakage and seizing of the teeth. And seizing of the teeth occurs under heavy load and calculation in checking for the possibility of tooth seizure is not effective. Hence, only calculations of the contacting fatigue strength and bending fatigue strength are carried out. 以点蚀、轮齿折断和胶合失效形式为主，其中胶合发生于重载，而且计算方法不成熟，所以一般只进行**齿面接触疲劳强度**和**齿根弯曲疲劳强度**计算。

(1) Low hardness gearing

软齿面齿轮传动：

The principal type of failure is pitting

以点蚀失效为主

- Designing on the basic of the calculating of contact fatigue strength of the working surfaces of the teeth
  按齿面接触疲劳强度设计
- Checking the bending fatigue strength of the teeth
  按齿根弯曲疲劳强度校核

❖ (2)High hardness gearing:

硬齿面齿轮传动

- Designing on the basic of the calculating of the bending fatigue strength of the teeth root
  按齿根弯曲疲劳强度设计
- Checking the contact fatigue strength of the working surfaces of the teeth
  按齿面接触疲劳强度校核

❖ The principal type of failure is tooth breakage

以轮齿折断失效为主

BACK

2、 Open gearing 开式齿轮传动:

The principal types of failure are abrasive wear and tooth breakage. Up till now, calculation for assessment of the service life of gears subject to substantial wear is not effective. Hence, the design is carried out on the basic of the calculations of the bending fatigue strength , then module m is increased by 10~15% to take into account the influence of wear.

以磨损和轮齿折断失效为主，其中磨损无可靠计算方法，所以按弯曲疲劳强度设计，然后考虑磨损因素，将模数m增加10%~15%。

§6-3 Loads in cylindrical gearing
圆柱齿轮传动载荷计算

☺Forces in spur cylindrical gearing

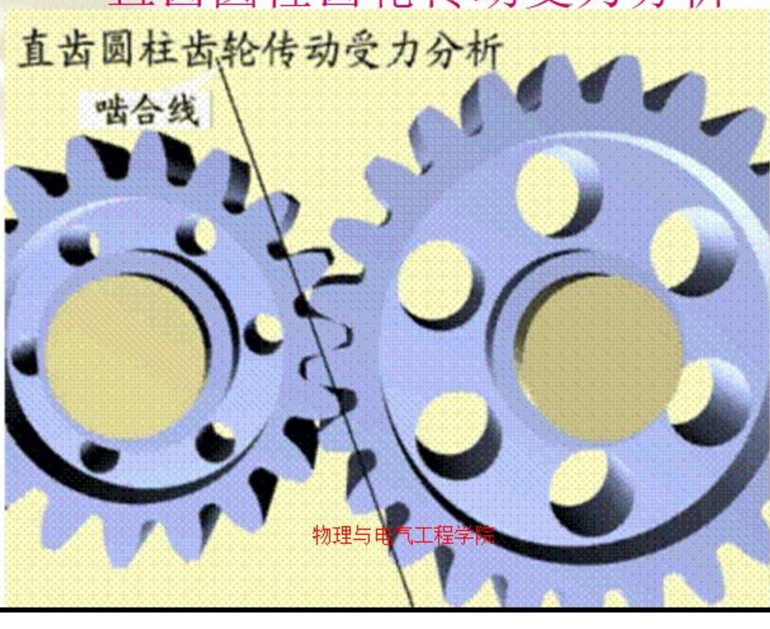
直齿圆柱齿轮传动受力分析

☺Forces in helical cylindrical gearing

斜齿圆柱齿轮传动受力分析

☺计算载荷

一、Forces in spur cylindrical gearing
直齿圆柱齿轮传动受力分析



1、Relationship between forces acting on driving and driven gears

1、各力间关系

$$\vec{F}_{n1} = -\vec{F}_{n2} \Rightarrow \begin{cases} \vec{F}_{t1} = -\vec{F}_{t2} \\ \vec{F}_{r1} = -\vec{F}_{r2} \end{cases}$$

2、力的大小 Magnitude of forces

$$\begin{cases} F_{t1} = F_{t2} = \dfrac{2T_1}{d_1}\left(= \dfrac{2T_2}{d_2}\right) \\ F_{r1} = F_{r2} = F_{t1} tg\ \alpha \\ F_{n1} = F_{n2} = F_{t1}/\cos\alpha = \dfrac{2T_1}{d_1 \cos\alpha} \end{cases}$$

3、Direction of forces 力的方向

①F_r :Directing from the pitch point to the rotating centre

由啮合点指向各自回转中心

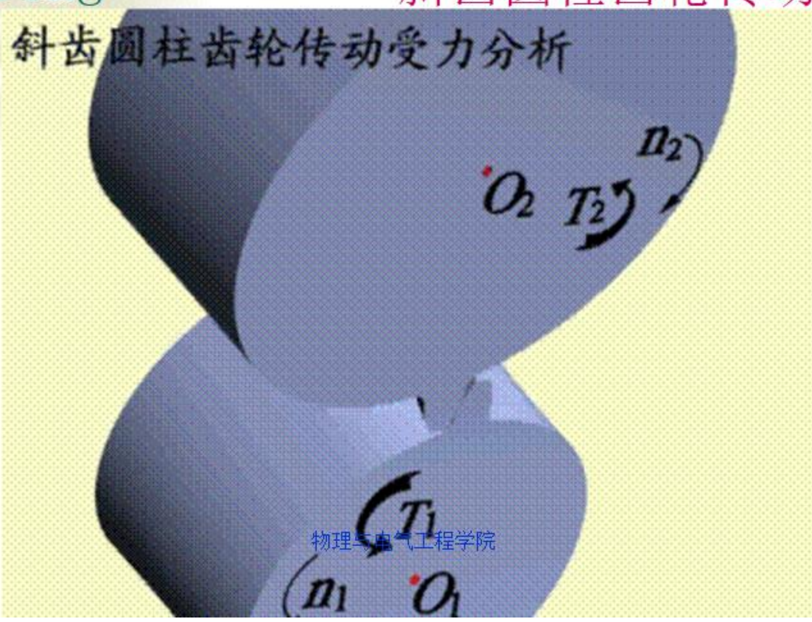
②F_t: 主动轮（阻力）：与回转方向相反

从动轮（驱动力）：与回转方向相同

Driving gear (resistance force): Its direction is opposite to the direction of rotation of driving gear.

Driven gear (driving force): Its direction is the same with the direction of rotation of driven gear.

二、 Forces in helical cylindrical gearing 斜齿圆柱齿轮传动受力分析



Relationship between forces acting on the driving and driven gears

1、各力间关系

$$\vec{F}_{n1} = -\vec{F}_{n2} \Rightarrow \begin{cases} \vec{F}_{t1} = -\vec{F}_{t2} \\ \vec{F}_{r1} = -\vec{F}_{r2} \\ \vec{F}_{a1} = -\vec{F}_{a2} \end{cases}$$

Resolve into

2、力的大小 Magnitude of forces

$$F_{t1}=F_{t2}=\frac{2T_1}{d_1}\left(=\frac{2T_2}{d_2}\right)$$

$$F_{r1}=F_{r2}=F_{t1}\operatorname{tg}\alpha_t=\frac{F_{t1}\operatorname{tg}\alpha_n}{\cos\beta}$$

$$F_{a1}=F_{a2}=F_{t1}\operatorname{tg}\beta$$

$$F_{n1}=F_{n2}=\frac{F_{t1}}{\cos\beta\cos\alpha_n}$$

$$=\frac{2T_1}{d_1\cos\beta\cos\alpha_n}=\frac{2T_1}{d_1\cos\beta_b\cos\alpha_t}$$

Helix direction: left hand for left-hand helix gear and right hand for right-hand helix gear.

3、力的方向 Direction of forces

F_r和F_t同前 As before

F_a：
- ①旋向：左旋伸左手，右旋伸右手
- ②回转方向：四指握拳方向
- ③F_a方向：
  - 主动轮：与拇指指向相同
  - 从动轮：与拇指指向相反

The opposite direction of thumbing

The same direction of thumbing

Rotating direction: the bending direction of four fingers

三、 Design load 计算载荷

$F_nd=KF_n$

K——Load factor 载荷系数

（Take into account the influence of multiform factors on the load 考虑各种因素对载荷实际影响）$K=K_AK_vK_\alpha K_\beta$

K_A ——Service factor 使用系数

Motor

原动机

Operating machine

工作机

Unsteady performance

性能不稳

Additional external dynamics

附加外部动载荷

Load grow instantly

载荷瞬时增大

K_A

K_v ——Dynamic load factor 动载系数

Errors in manufacture and assembly 制造安装误差

Deflections of the teeth under load 轮齿变形

Errors in engagement 啮合误差

Instantaneous angular acceleration 瞬时角加速度:

Additional internal dynamic load 附加内部动载荷

Load grow instantly 载荷瞬时增大

K_v

Decrease in velocity, increase in accuracy of gear manufacture 速度v降低,精度增高

Modification of the tooth profile face 齿顶修形

Decrease in dynamic load 动载荷减小

K_v Drops

K_α——Factor of load distribution among the teeth

齿间载荷分配系数

Errors in manufacture

由于误差

载荷在各啮合齿对间分配不均

No uniform load distribution among the teeth

某对齿所受载荷高于平均值

Load acting on some gear is more than the average load

K_α

齿轮啮合时，由于存在单对齿与双对齿啮合，轮齿变形及齿轮制造误差等原因，载荷在两啮合齿对之间的分配是不均匀的。在计算齿轮强度时，用齿间载荷分配系数考虑其影响。

齿轮传动的啮合过程

啮合中轮齿的变形曲线

轮齿刚度对Kα的影响

基圆齿距误差对Kα的影响

K_{β} —— Factor of the nonuniform loading along the face widt

齿向载荷分配系数

Bending and torsion deformation of the shaft

轴弯曲、扭转变形

Elastic displacement of bearings

轴承弹性位移

Errors in manufacture and assembly

制造安装误差

齿轮副相倾斜、轮齿扭曲

The gears are in misalignment with respect to each other

The load is not uniformly distributed along the face width

载荷沿接触线分布不均

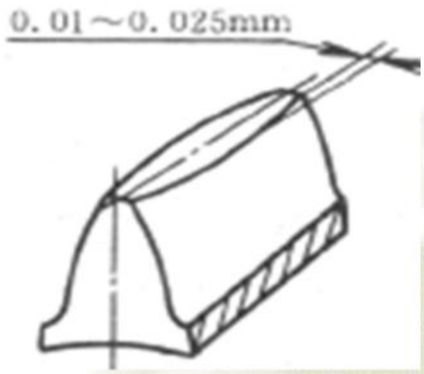
Load acting on some part of the teeth along the face width increase

载荷局部增大

K_{β}

Load concentration can be reduced by:

① Making the teeth with a tapered crown 鼓形齿



②Proper face width

齿宽合理

③ Locating the gears favorably between the bearings

齿轮布置合理

④Increasing the rigidity of the shaft and bearings

增加轴和轴承座刚度

⑤Raising the accuracy of manufacture and assembly

提高制造、安装精度

由上述五点 →

Load concentration is reduced

偏载降低

→ K_β Decreased

齿轮工作时，由于轴的弯曲变形和扭转变形以及传动装置的制造和安装误差等原因，使轮齿沿接触线产生载荷分布不均匀现象，在计算齿轮强度时，用齿向载荷分布系数考虑其影响。

Influence of bending deformation of the shaft on K_β

轴弯曲变形对$K\beta$的影响

Influence of torsion deformation of the shaft on K_β

轴扭转变形对$K\beta$的影响

Influence of total deformation of the shaft on K_β

弯曲扭转综合变形的影响

§6-4 Strength calculations for spur gears 直齿圆柱齿轮传动强度计算

- Contact fatigue strength calculation 齿面接触疲劳强度计算
- Beam fatigue strength calculation 齿根弯曲疲劳强度

BACK

一、 Contact fatigue strength calculation 齿面接触疲劳强度计算

1、 Formulas 公式推导

①The Hertz formula and strength condition ：赫兹公式与强度条件

$$\sigma_H = \sqrt{\frac{F}{\pi L} \frac{\frac{1}{\rho}}{\left(\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}\right)}} \le [\sigma]_H \text{......................} ①$$

$$\frac{1}{\rho} = \frac{1}{\rho_1} \pm \frac{1}{\rho_2}$$

② Characteristic point 计算特征点：

Pitch point 节点（ Due to pitting begins at the line passing through this point where load is transmitted by single pair of teeth. Another reason is to simplify calculation.

此处为单对齿啮合，最易发生点蚀；此处计算过程简化.）

③ Equivalent radius of curvature 综合曲率半径 ρ

The contact stress depend on load and the radii of curvature of the contact point of teeth. And the contact stress varies along with the change of the contact point.

齿面接触应力的大小与载荷、接触点的综合曲率半径等密切相关，且啮合点的接触应力随啮合点的位置改变而变化。

Radius of curvature of gear

齿廓曲率半径

Equivalent radius of curvature

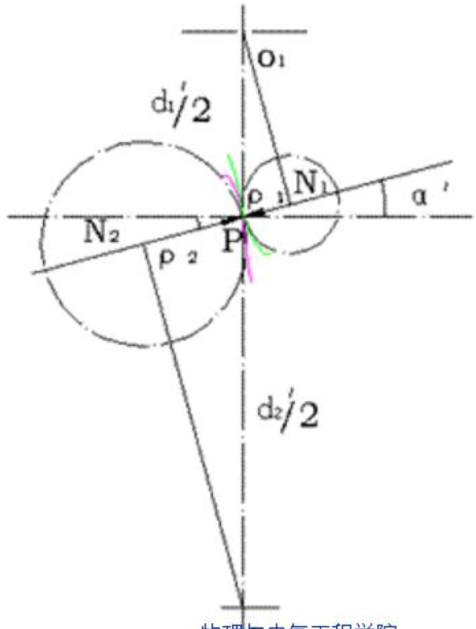
综合曲率

综合曲率的影响

Influences of radius of curvature

接触应力变化规律

Changes of contact stress



④ Length of the line of contact 接触线长度 L

Face width of tooth 齿宽b

$$L = b / Z_\varepsilon^2 \text{ .... } ③$$

$$Z_\varepsilon = \sqrt{\frac{4-\varepsilon_\alpha}{3}} \text{......重合度系数} \qquad \varepsilon_\alpha \text{......端面重合度}$$

Contact ratio factor

The face contact ratio

⑤ Load

$$F = KF_n = \frac{2KT_1}{d_1 \cos\alpha} \text{ .... } ④$$

⑥ Substituting ②、③、④ into ①

$$\sigma_H = \sqrt{\frac{1}{\pi\left(\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right)}} \sqrt{\frac{2}{\cos^2\alpha tg\alpha'}} Z_\varepsilon \sqrt{\frac{2KT_1}{bd_1^2}\frac{(u\pm1)}{u}} \le [\sigma]_H$$

⇓ 弹性系数Z_E ⇓ 节点区域系数Z_H

Factor taking into account the mechanical properties of the gear materials

Factor taking into account the shape of the contacting surfaces

$$\sigma_H = Z_E Z_H Z_\varepsilon \sqrt{\frac{2KT_1}{bd_1^2}\frac{(u\pm1)}{u}} \le [\sigma]_H$$

（校核公式）

For checking

⑦

设 $b/d_1 = \psi_d$ 齿宽系数　　则 $b = \psi_d d_1$ 代入

Face width factor

It is substituted in following equation

$$\sigma_H = Z_E Z_H Z_\varepsilon \sqrt{\frac{2KT_1}{\psi_d d_1^3}\frac{(u\pm1)}{u}} \le [\sigma]_H$$

We obtain:

$$d_1 \ge \sqrt[3]{\frac{2KT_1}{\psi_d}\frac{(u\pm1)}{u}\left(\frac{Z_E Z_H Z_\varepsilon}{[\sigma]_H}\right)^2}$$

For design

（设计公式）

二、 Beam fatigue strength of tooth root

齿根弯曲疲劳强度

1、 Formula

1） Basic hypotheses 基本假设

① Gear tooth is treated as a cantilever and the whole force is applied to the top of the tooth

轮齿视为悬臂梁，全部载荷作用于齿顶

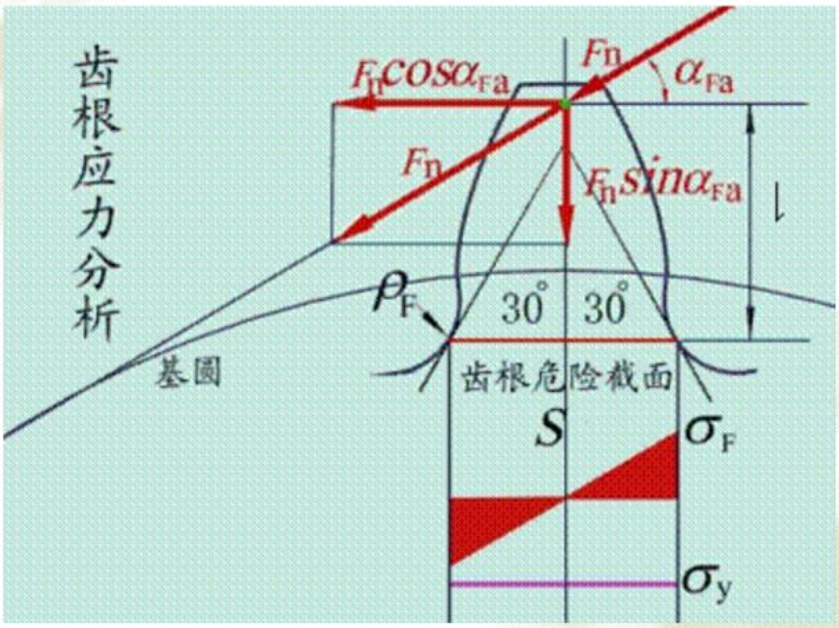
②The strength is determined for the side of the teeth subject to tensile stress

以受拉边为计算依据

③ The dangerous cross section is taken by30° tangent to the axis of the tooth 危险截面AB（30° 切线法）

④Only considering peripheral component of the force (compression stress caused by vertical component less than 10% of bending stress)

仅计水平力$F_n\cos\alpha_F$（σ仅为σ_b百分之几）



2） Introduction 推导过程

① On the dangerous cross section：

危险截面处

$$\sigma_F = \sigma_{bmax} = \frac{m}{W}$$

$$m = F_n \cos\alpha_F l = F_t \frac{\cos\alpha_F}{\cos\alpha} l \qquad W = \frac{bS^2}{6}$$

$$代入：\sigma_F = \frac{F_t}{\frac{bS^2}{6}} \frac{\cos\alpha_F}{\cos\alpha} l = \frac{2T_1}{d_1 bm} \frac{6\left(\frac{l}{m}\right)\cos\alpha_F}{\left(\frac{s}{m}\right)^2 \cos\alpha}$$

$$\Downarrow$$

齿形系数Y_{Fa}

$$\sigma_F = Y_{Fa} \frac{2T_1}{d_1 bm}$$

Tooth form factor

② 计入

Load factor 载荷系数K

Stress concentration factor

应力修正系数Y_{Sa}(应力集中）

Contact ratio factor

重合度系数Y_ε

（Load is transmitted by more than one pair of teeth 载荷不只一对齿承担）

$$\sigma_F = K Y_{Sa} Y_\varepsilon \sigma_{F_0} = \frac{2KT_1}{bd_1m} Y_{Fa} Y_{Sa} Y_\varepsilon \le [\sigma]_F$$ （校核公式）

For checking

将 $b=\Psi_b d_1$ 及 $d_1=mZ_1$ 代入：

Substituting b and d_1 into above formula:

$$\sigma_F = \frac{2KT_1}{\Psi_b m^3 Z_1^2} Y_{Fa} Y_{Sa} Y_\varepsilon \le [\sigma]_F$$

$$m \ge \sqrt[3]{\frac{2KT_1}{\Psi_b Z_1^2}\left(\frac{Y_{Fa} Y_{Sa} Y_\varepsilon}{[\sigma]_F}\right)}$$ （设计公式）

For design

2、 Notes

① Module increases 模数m增大

σ_F decrease

Bending strength mainly depend on m.

弯曲强度主要取决于m

（For power drives:动力传动m≥1.5～2mm）

② Contact ratio fac

$$Y_\varepsilon = 0.25 + \frac{0.75}{\varepsilon_a}$$

③ Tooth form factor Y_{Fa} is independent of m, only depend on tooth form

齿形系数Y_{Fa}：P59 图3-21 与模数m无关，仅取决于齿形（Z，x）

$Z\downarrow \rightarrow Y_{Fa}\uparrow \rightarrow \sigma_{F1} > \sigma_{F2}$

Usually $[\sigma]_{F1} > [\sigma]_{F2}$

* Bending strength of the pinion and gear should be calculated respectively or carried out for the toothed gear with the higher $\dfrac{Y_{Fa}Y_{Sa}}{[\sigma]_F}$ of

大小齿轮弯曲强度应该分别计算，或取 $\dfrac{Y_{Fa}Y_{Sa}}{[\sigma]_F}$ 中大者计算。

④ $[\sigma]_F$—— Allowable bending stress

许用弯曲应力 σ_{Flim}——Bending endurance limit of tested gears with 1% of fatigue possibility

失效概率为1%时，试验齿轮弯曲疲劳强度 P67 图3－28.

S_F——Safety margin 安全系数（一般S_F＝1）

K_{FN}——Service life factor 寿命系数 K_{FN}＝f（N） P65 图3－26

$$[\sigma]_F = \frac{\sigma_{Flim} K_{FN}}{S_F}$$

⑤ Z_1（ Number of the teeth of the pinion 小齿轮齿数）

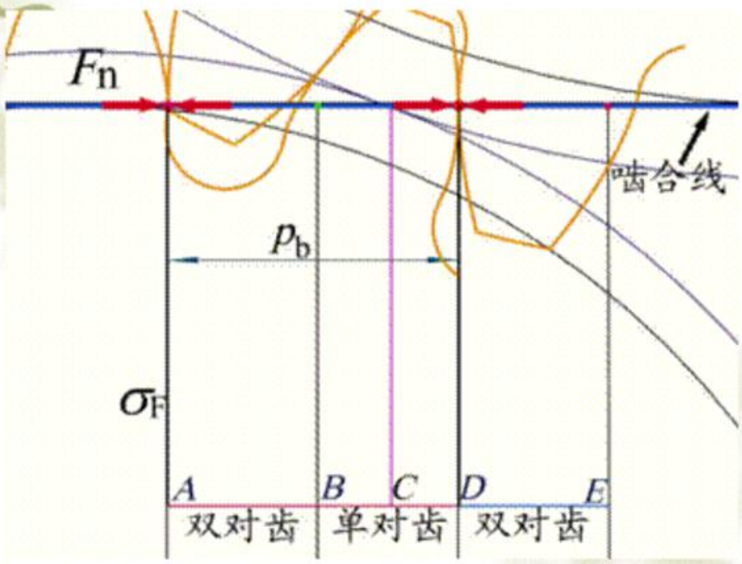
Low hardness close gearing: (Overall size of the drive mainly depends on contact strength, while bending strength of the drive is sufficient.)

闭式软齿面齿轮：（传动尺寸主要取决于接触强度，弯曲强度较富裕）

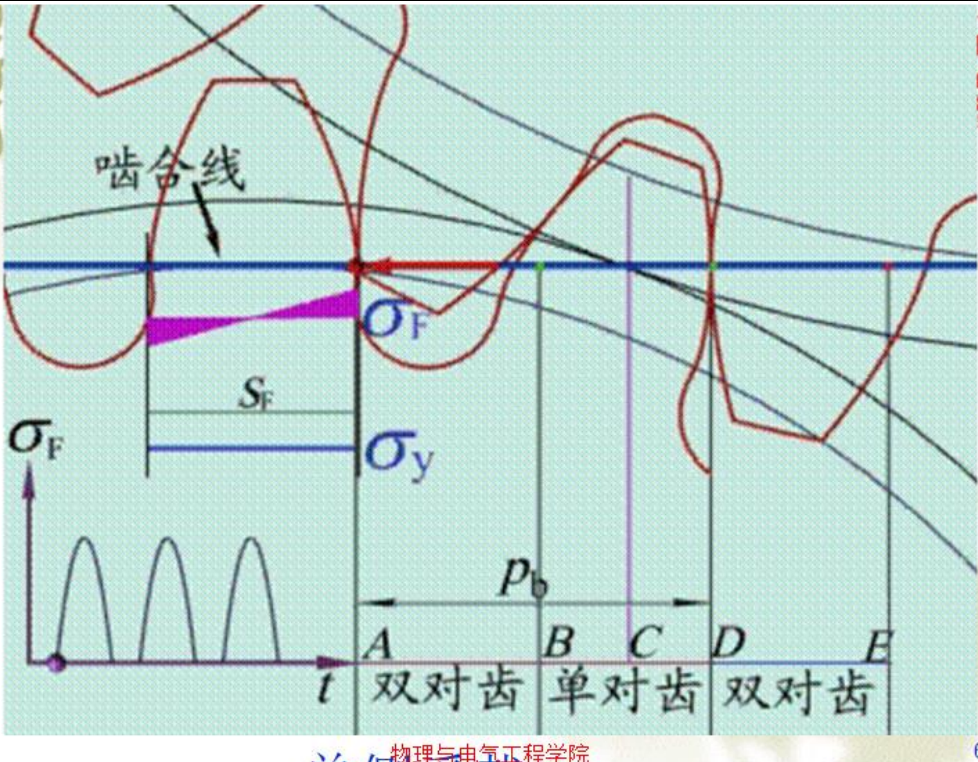
When d_1=const and bending strength is ensured, it is suggested that larger number of the teeth of the pinion should be taken.

在d_1=const时，并满足弯曲强度前提下，尽量取多的齿数

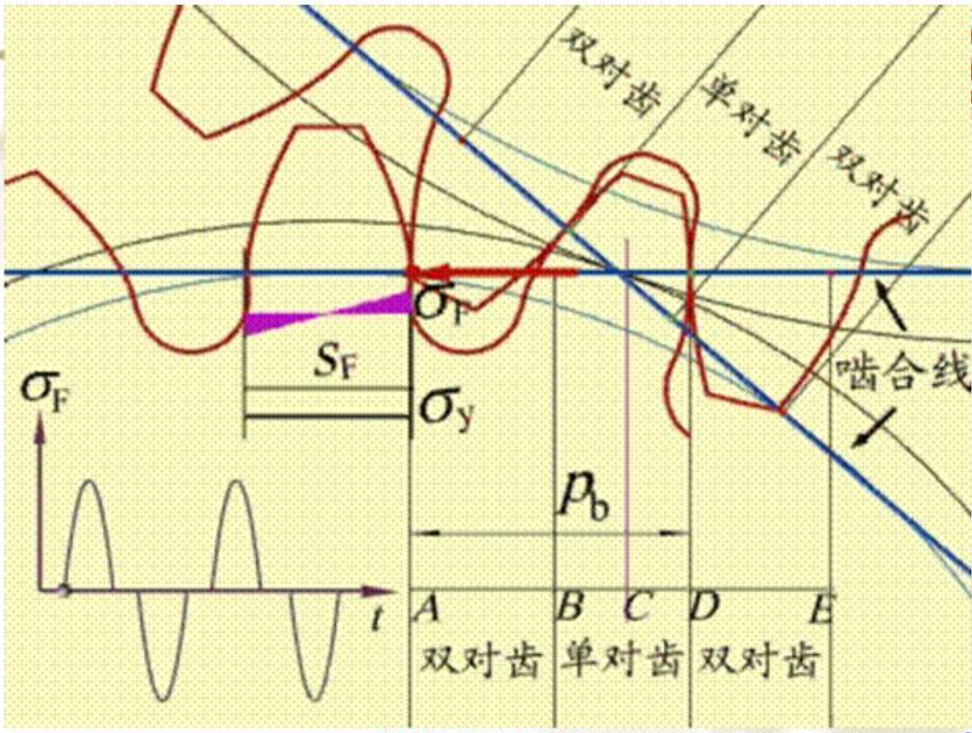
（Z_1↑，m相应↓， ∵ $d_1=mZ_1$）



齿根弯曲应力变化规律



单侧受载



双侧受载

§6-5 Strength for helical cylinder gearing 斜齿圆柱齿轮传动强度计算

- Contacting fatigue strength of the tooth surface

  齿面接触疲劳强度计算

- Bending strength

  弯曲强度计算

BACK

一、Contacting fatigue strength of the tooth surface　齿面接触疲劳强度计算

1、 Formula

按节点处法面当量直齿圆柱齿轮计算。

$$\sigma_H = \sqrt{\frac{F}{\pi L}\frac{\frac{1}{\rho}}{\left(\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right)}} \le [\sigma]_H \text{..........................} ①$$

① $\frac{1}{\rho}=?$

$$\rho_{n1}=\frac{\rho_{t1}}{\cos\beta_b}=\frac{d_1'\sin\alpha_t'}{2\cos\beta_b}$$

$$\rho_{n2}=\frac{\rho_{t2}}{\cos\beta_b}=\frac{d_2'\sin\alpha_t'}{2\cos\beta_b}$$

$$\frac{1}{\rho}=\frac{\rho_{n2}/\rho_{n1}\pm1}{\rho_{n1}\frac{\rho_{n2}}{\rho_{n1}}}=\frac{2\cos\beta_b}{d_1'\sin\alpha_t'}\bullet\frac{u\pm1}{u}\ \text{...②}$$

$$\frac{\rho_{n2}}{\rho_{n1}}=\frac{d_2'}{d_1'}=\frac{d_2}{d_1}=\frac{Z_2}{Z_1}=u$$

图 5-23 斜齿圆柱齿轮传动节点的曲率半径

② The length of the line of

接触线长度L（ Varies）

$$取L=L_{min}=\frac{b}{Z_\varepsilon^2\cos\beta_b}\ \text{......③}$$

Axial contacting ratio

$$Z_\varepsilon=\sqrt{\frac{4-\varepsilon_\alpha}{3}(1-\varepsilon_\beta)+\frac{\varepsilon_\beta}{\varepsilon_\alpha}}$$

ε_β......轴向重合度

$\varepsilon_\beta\ge1$时，取$\varepsilon_\beta=1$

则$Z_\varepsilon=\sqrt{\frac{1}{\varepsilon_\alpha}}$

③

F

$$F = KF_n = \frac{2KT_1}{d_1}\frac{1}{\cos\alpha_t \cos\beta_b} \cdots\cdots④$$

④ Substituting ②、 ③ 、④代入into ①， and favorable influence of the line of contact inclined to the root of the teeth on the contacting strength of the gears is considered by helix anger factor

此外，计入螺旋角系数以考虑接触线倾斜对接触强度的有利影响

$$Z_\beta = \sqrt{\cos\beta}$$

$$\sigma_H = \sqrt{\frac{1}{\pi\left(\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right)}}\sqrt{\frac{2\cos\beta_b}{\cos^2\alpha_t \, tg\alpha_t'}}Z_\varepsilon\sqrt{\frac{2KT_1}{bd_1^2}\frac{(u\pm1)}{u}}Z_\beta \le [\sigma]_H$$

$\Downarrow$ 弹性系数Z_E

$\Downarrow$ 节点区域系数Z_H

Factor taking into account the mechanical properties of the gear materials

Factor taking into account the shape of the contacting surfaces

$$\sigma_H = Z_E Z_H Z_\varepsilon Z_\beta \sqrt{\frac{2KT_1}{bd_1^2}\frac{(u\pm1)}{u}} \le [\sigma]_H$$

（校核公式）

For checking

$b = \psi_d d_1$代入

$$d_1 \ge \sqrt[3]{\frac{2KT_1}{\psi_d}\frac{(u\pm1)}{u}\left(\frac{Z_E Z_H Z_\varepsilon Z_\beta}{[\sigma]_H}\right)^2}$$

For design

（设计公式）

2、Notes

① Helical gears are stronger than spur gears for the following reason
斜齿轮相对于直齿轮承载能力↑：

$$\because \begin{cases} \rho_n \uparrow \\ L \uparrow \end{cases}$$

BACK

② β

β↓ Advantages is not obvious
优点不明显

β↑ F_a↑→ Service life of the bearing↓
轴承寿命

Usually β＝8°～25°
(Herringbone gears: β＝20°～35
)

二、 Bending strength 弯曲强度计算

1、 Formula ：

Helical teeth can be assumed to conformed with the tooth of a equivalent spur gear in a cross section normal to the tooth. The strength calculation is done against the equivalent spur gear.

按法面当量直齿圆柱齿轮计算

BACK

① Basic parameters of the equivalent spur gear

当量齿轮的基本参数：

Module m_n、

Equivalent number of teeth 当量齿数

$$Z_{v1} = Z_1 / \cos^3\beta \qquad Z_{v2} = Z_2 / \cos^3\beta$$

②Favorable influence of the line of contact inclined to the root of the teeth on the bending strength of the gears is taken into account by helix anger factor Y_β

引入螺旋角系数Y_β考虑接触线倾斜对弯曲强度的有利影响

By analogy with spur gearing:

采用比较法

直齿轮：$\sigma_F = \frac{2KT_1}{bd_1m} Y_{Fa} Y_{Sa} Y_\varepsilon \le [\sigma]_F$

Spur gear

斜齿轮：$\sigma_F = \frac{2KT_1}{bd_1m_n} Y_{Fa} Y_{Sa} Y_\varepsilon Y_\beta \le [\sigma]_F$ （校核公式）

Helical gear

For checking

Substituting

$$b=\psi_d d_1 \quad d_1=\frac{m_n Z_1}{\cos\beta}$$ 代入：

into above formula:

$$m_n \ge \sqrt[3]{\frac{2KT_1\cos^2\beta}{\Psi_b Z_1^2}\left(\frac{Y_{Fa}Y_{Sa}Y_\varepsilon Y_\beta}{[\sigma]_F}\right)}$$

For design （设计公式）

2、 Notes

①

$$Y_\beta=1-\varepsilon_\beta\frac{\beta^\circ}{120^\circ}$$ Helix anger factor 螺旋角系数 $\begin{cases}\varepsilon_\beta \ge 1 & 取\varepsilon_\beta=1\\ Y_\beta<0.75 & 取Y_\beta=0.75\end{cases}$

$$Z_V=\frac{Z}{\cos^3\beta}$$

② Y_{Fa}、 Y_{Sa} Taken by equivalent number of teeth 据当量齿数查取

③ Helical gears are stronger than spur gears in bending strength 斜齿轮和直齿轮相比，弯曲强度增大.

（$Y_{\beta}<1$，$Z_v\uparrow \rightarrow Y_{Fa}\downarrow \rightarrow \sigma_F\downarrow$）

§6-6 Spur bevel gearing drive 直齿圆锥齿轮传动 (Usually $\Sigma=90^\circ=\delta_1+\delta_2$)

- Special features of the bevel gearing as compared with cylindrical gearing 和圆柱齿轮相比，锥齿轮的特点
- Geometrical relationship and basic parameters of the equivalent gear for bevel gearing

  几何关系和当量齿轮基本参数
- Forces in bevel gearing 受力分析
- Contact strength 接触强度
- Bending strength 弯曲强度
- Notes 说明
- Problems 思考题

BACK

一、 Special features of the bevel gearing as compared with cylindrical gearing

和圆柱齿轮相比，锥齿轮的特点：

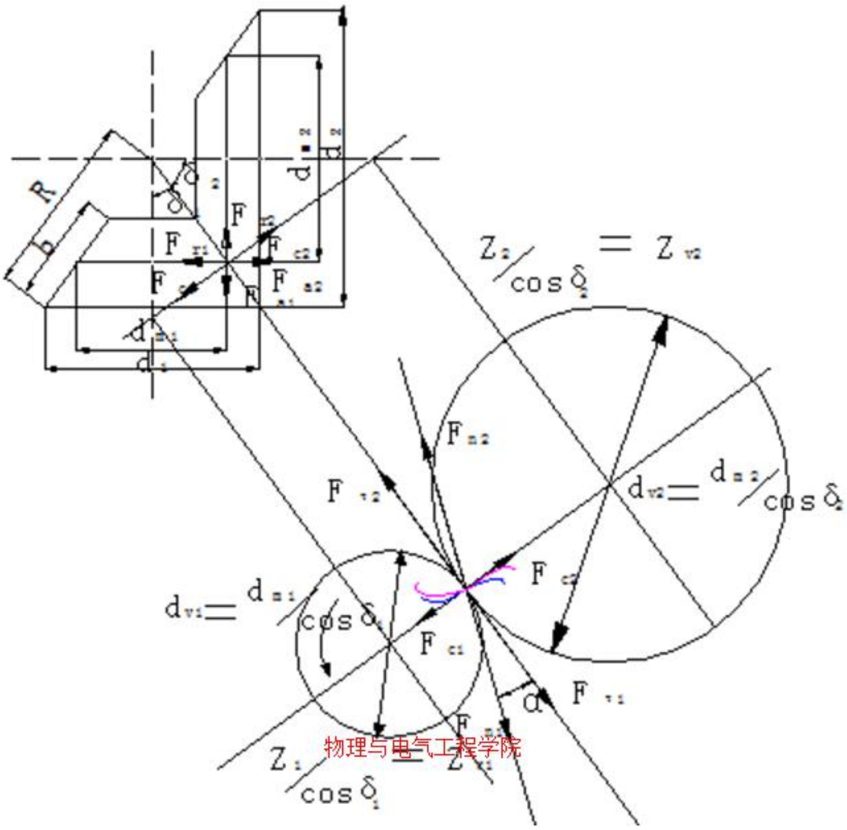
① Lower machining accuracy ,Higher level of noise Suitable for low peripheral velocities, Outside diameter of bevel gears should not be large.

制造精度低,振动噪音大，圆周速度不宜过高，大锥齿轮直径不宜过大。

② The tooth dimensions of bevel gears are specified at the large end on the back cone because it is more convenient to make measurements here

为了检验和确定轮廓尺寸方便，规定大端参数为标准值；

③ For strength calculations, the bevel gears are replaced by spur gears of a pitch diameter and module equal to those at the middle cross section of the bevel gears 强度计算转化为锥齿轮齿宽中点处当量直齿圆柱齿轮进行。



二、Geometrical relationship and basic parameters of the equivalent gear for bevel gearing 几何关系和当量齿轮基本参数

1、 Geometrical relationships 几何关系

$$
\begin{cases}
tg\delta_1 = \dfrac{\left(d_1/2\right)}{\left(d_2/2\right)} = \dfrac{1}{u} \\[2ex]
tg\delta_2 = \dfrac{\left(d_2/2\right)}{\left(d_1/2\right)} = u
\end{cases}
\qquad
\begin{cases}
\cos\delta_1 = \dfrac{1}{\sqrt{1+tg^2\delta_1}} = \dfrac{u}{\sqrt{1+u^2}} \\[2ex]
\cos\delta_2 = \dfrac{1}{\sqrt{1+tg^2\delta_2}} = \dfrac{1}{\sqrt{1+u^2}}
\end{cases}
$$

Face width factor 齿宽系数

$\Psi_R = b/R \le 1/3$

Usually 0.25～0.3

$d_m = d\,(1 - 0.5\,\Psi_R)$

2、 Basic parameters

基本参数

$$m_v = m_m = \frac{d_m}{Z} = m(1-0.5\psi_R)$$

$$d_v = d_m/\cos\delta \rightarrow m_v Z_v = m_m Z/\cos\delta \rightarrow Z_v = Z/\cos\delta$$

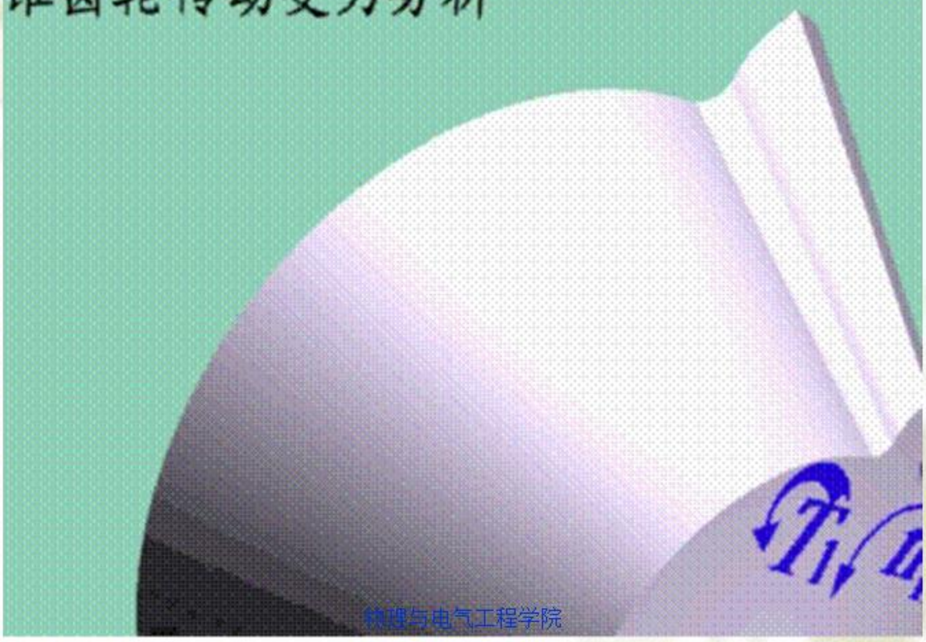
三、 Forces in bevel gearing 受力分析

1、 Relationship between forces 各力间关系

Acting on driving and driven gears:

$$\vec{F}_{t1} = -\vec{F}_{t2} \qquad \vec{F}_{r1} = -\vec{F}_{a2} \qquad \vec{F}_{a1} = -\vec{F}_{r2}$$

锥齿轮传动受力分析



2、Magnitude of forces 力的大小：

$$F_{t1} = F_{t2} = {2T_1}/{d_{m1}}$$

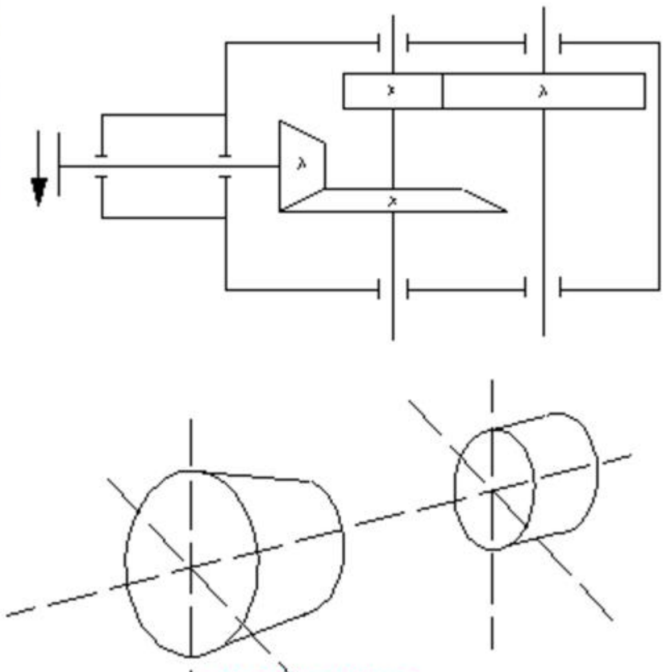
$$F_{r1} = F_{a2} = F_{c1}\cos\delta_1 = F_{t1}\,tg\alpha\cos\delta_1$$

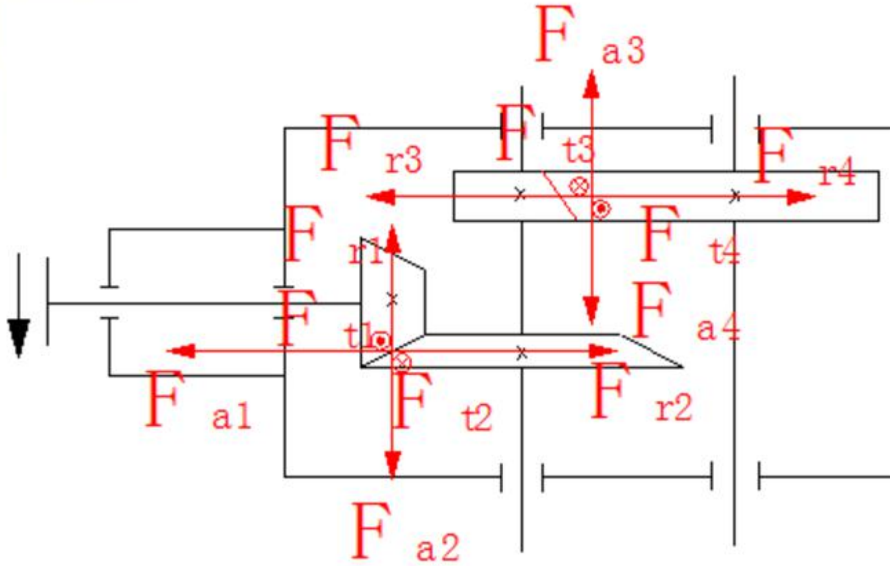
$$F_{a1} = F_{r2} = F_{c1}\sin\delta_1 = F_{t1}\,tg\alpha\sin\delta_1$$

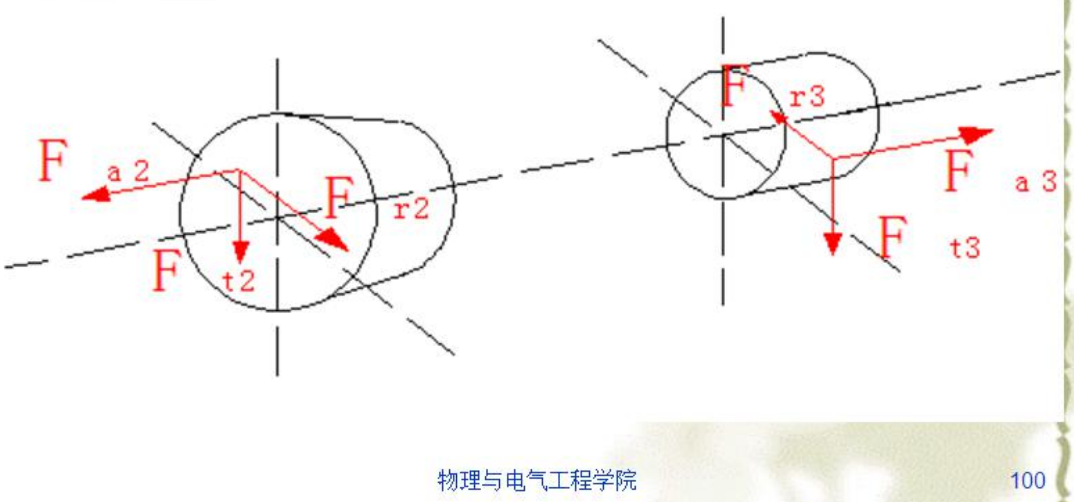
$$F_{n1} = F_{n2} = {F_{t1}}/{\cos\alpha} = {2T_1}/{d_{m1}\cos\alpha}$$

3、 Directions of forces 力的方向：

- F_r、F_t 同前
- F_a Toward the large end 指向大端




F a3
F r3
F t3
F r4
F r1
F t4
F t1
F a4
F a1
F t2
F r2
F a2


F a 2
F r2
F t2
F r3
F a 3
F t3

四、 Contact strength 接触强度

$$\sigma_H = Z_E Z_H Z_\varepsilon \sqrt{\frac{2KT_{v1}}{bd_{v1}^2}\frac{(u_v+1)}{u_v}} \le [\sigma]_H$$

① Due to lower manufacture accuracy, factor Z_ε is neglected

制造精度低，所以忽略Z_ε

②

$$d_{v1} = d_{m1}/\cos\delta_1 = (1-0.5\psi_R)d_1\frac{\sqrt{1+u^2}}{u}$$

③

$$u_v = Z_{v2}/Z_{v1} = \frac{Z_2}{Z_1}\bullet\frac{\cos\delta_1}{\cos\delta_2} = u/tg\delta_1 = u^2$$

④

$$T_{v1}=F_{t1}\frac{d_{v1}}{2}=\frac{2T_1}{d_{m1}}\frac{d_{v1}}{2}=\frac{2T_1}{d_{v1}\cos\delta_1}\frac{d_{v1}}{2}$$

$$=\frac{T_1}{\cos\delta_1}=T_1\frac{\sqrt{1+u^2}}{u}$$

⑤

$$b=\psi_R R=$$

$$\psi_R\frac{d_1}{2\sin\delta_1}=\psi_R\frac{d_1}{2}\sqrt{1+u^2}$$

❖Thus

$$\sigma_H=Z_EZ_H\sqrt{\frac{4KT_1}{\psi_R(1-0.5\psi_R)^2ud_1^3}}\le[\sigma]_H \quad \text{(校核公式)}$$

$$d_1\ge\sqrt[3]{\frac{4KT_1}{\psi_R(1-0.5\psi_R)^2u}\left(\frac{Z_EZ_H}{[\sigma]_H}\right)^2} \quad \text{(设计公式)}$$

For design

五、 Bending strength 弯曲强度

$$\sigma_F = \frac{2KT_{v1}}{bd_{v1}m_v} Y_{Fa} Y_{Sa} Y_{\varepsilon} \le [\sigma]_F$$

① Due to lower manufacture accuracy, factor Y_{ε} is taken as 1. 制造精度低，Y_{ε}取为1，

② T_{v1}、d_{v1}同前，$m_v = m_m = m(1-0.5\psi_R)$

$$\sigma_F = \frac{4KT_1}{\Psi_R(1-0.5\Psi_R)^2 Z_1^2 m^3 \sqrt{1+u^2}} Y_{Fa} Y_{Sa} \le [\sigma]_F \quad \text{(校核公式)}$$

$$m \ge \sqrt[3]{\frac{4KT_1}{\Psi_R(1-0.5\Psi_R)^2 Z_1^2 \sqrt{1+u^2}} \left(\frac{Y_{Fa} Y_{Sa}}{[\sigma]_F}\right)} \quad \text{(设计公式)}$$

六、 Notes

①Y_{Fa}、Y_{Sa} 根据 $Z_v = Z/\cos\delta$ 查取 Y_{Fa}、Y_{Sa} Are taken from $Z_v = Z/\cos\delta$

②$K = K_A K_v K_\beta$ （Factor K_α related to ε is neglected 不计与ε有关的K_α）

$K_v = f(v_m Z_1/100)$

$K_\beta = f(\psi_{dm})$

$$\Psi_{dm} = b/d_{m1} = \frac{\Psi_R\sqrt{1+u^2}}{2-\Psi_R}$$