



第十章 Shafts and axles 轴

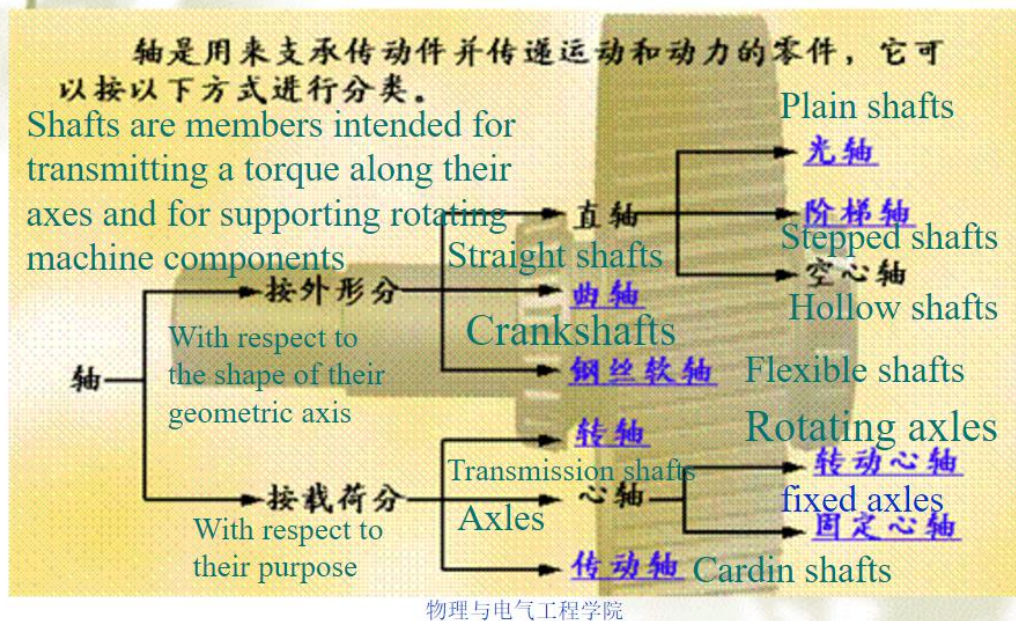
- § 10-1 General considerations 概述
- § 10-2 Structural design 结构设计
- § 10-3 Strength calculations 轴的强度校核
- § 10-4 Vibration calculations and critical speed
轴的振动计算和临界转速



§ 10-1 General considerations 概述

- ❖ Types 分类
- ❖ The main design problems
轴设计主要问题

Types 分类



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二、The main design problems 轴设计主要问题

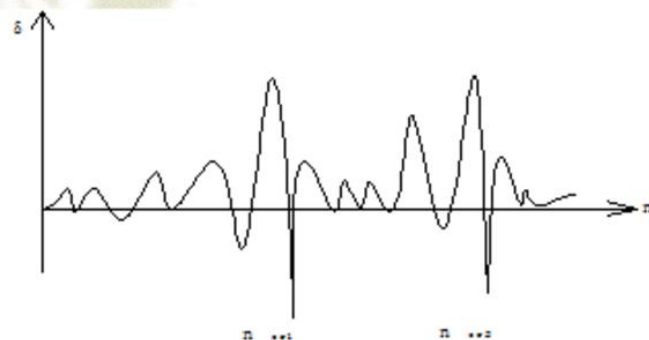
- ① Selection of the materials 选材
- ② Structural design (shape and sizes)
结构设计 (外形、尺寸)
- ③ Strength calculations 强度计算
(For fatigue failures and permanent deformation
防断裂、塑性变形)
- ④ Rigidity calculations 刚度计算
(For large elastic displacement 防过大变形)
- ⑤ Vibration stability calculations 振动稳定性计算
(For resonance of the high speed shaft 高速轴防共振)

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1、 Vibration stability calculations 关于振动稳定性计算



or:

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$$\begin{cases} n \leq (0.75 \sim 0.8) n_{cr1} \\ 1.4 n_{cr1} \leq n \leq 0.7 n_{cr2} \end{cases}$$



2、 Rigidity calculations 关于轴的刚度计

Shaft in operation:

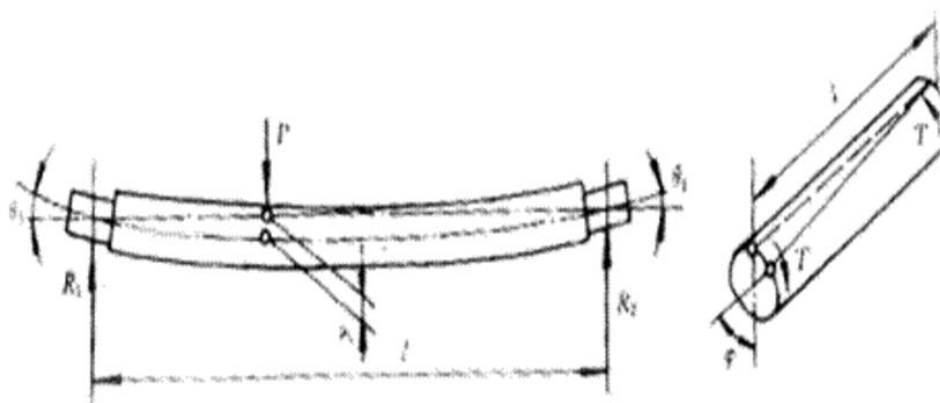
Deflections 挠度 y
Angles of inclination
转角 θ
Angles of twist 扭角 ϕ

Angular displacement of meshing gears with respect to each other and consequent load concentration along the face width of the meshing teeth

齿轮偏载和轴承载荷集中

$$\therefore y \leq [y] \quad \theta \leq [\theta] \quad \phi \leq [\phi]$$

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3、Selection of the shaft materials



关于轴的材料选择

- 1) Carbon steels 碳钢
- ① Common carbon steels:
Q235~Q275
For light load shafts
普通碳素钢：不重要或者受力较小
 - ② Good-quality carbon steels: 45、50
For common shaft
(structural improvement, normalization)
优质碳素钢 一般 (正火)

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2) Alloy steels 合金钢

20Cr、40Cr等 For heavy load shafts or when there are strict requirements as to weight, or requirements as to high wear resistance

(Hardening Structural improvement)

重载或者要求减轻重量时或要求提高耐磨性时
(淬火、调质)



❖3) Cast iron

Nodular cast iron, alloy cast iron
Shaped shafts

铸铁 QT、合金铸铁
形状复杂轴



★ Modulus of elasticity of the steels is not nearly depend on their grades and heat treatments. Changes of the grades and heat treatments of steels are invalid increase of the rigidity of the shaft. Effective methods are increase of the diameter and decrease of the length of the shaft.

钢种和热处理对弹性模量影响很小，对增大轴刚度无效（增大轴径、减小跨距）

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★ Alloy steels have high strength, but they have high susceptibility to stress concentration. When surfaces of the shafts are roughly machined or their structures are not reasonably designed, the endurance limit of alloy steels turns out to be no higher than that of ordinary medium-carbon steels.

合金钢强度高，但对应力集中敏感，如果结构上和表面上不注意，轴强度得不到改善

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§ 10-2 Structural design 结构设计

Given data 已知:

Find 求:

- ① Power transmitted by shaft 、
speed of the shaft

轴传递的功率 P 、转速 n
(Torque 转矩 T)

- ② Dimensions of the driving
elements 传动件尺寸

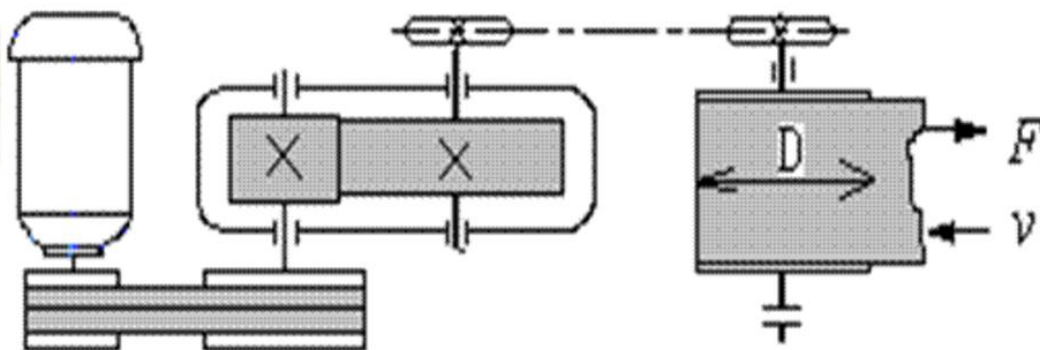
- ③ Scheme of assembly 装配简图

- ① Reasonable shape of
shaft

轴的合理外形

- ② All dimensions of
shaft 轴全部尺寸

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带式运输机传动简图



一、Tentative calculations 初算:

Strength of the shaft is checked after structural design is completed.

$$\left. \begin{array}{l} 9.55 \times 10^6 \frac{P}{n} \Rightarrow T \\ \text{支点未知} \Rightarrow M? \end{array} \right\} \text{先结构设计, 后强度校核}$$

Length of the shaft has not yet been determined.



Usually, the end of shaft do not bear bending moment or bear very limited bending moment. Hence, the end diameter of shaft can be estimated according to torque T .

d_{\min} 处一般没有弯矩 M

或者弯矩 M 很小, 可以根据扭矩 T 估算 d_{\min}

carried out by conditional calculations of the shaft in torsion

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$$\tau = \frac{T}{W_T} = \frac{9.55 \times 10^6 \frac{P}{n}}{0.2 d_{\min}^3} \leq [\tau] \quad (\text{MPa})$$

$$d_{\min} \geq \sqrt[3]{\frac{9.55 \times 10^6 \frac{P}{n}}{0.2 [\tau]}} = C \sqrt[3]{\frac{P}{n}} \quad (\text{mm})$$

$C = \begin{cases} d_{\min} \text{ 处无弯矩 } M: \text{ 取小值} \\ \text{(带轮} > \text{链轮} > \text{联轴器)} \\ d_{\min} \text{ 处有弯矩 } M: \text{ 取大值} \\ \text{(减小 } [\tau], \text{ 即增大 } C, \text{ 考虑弯矩的影响)} \end{cases}$

When the end of shaft does not bear bending moment factor C take smaller value when the end of shaft bears bending moment factor C take larger value

Since the bending is not taken into account, a lower value of $[\tau]$ is used.

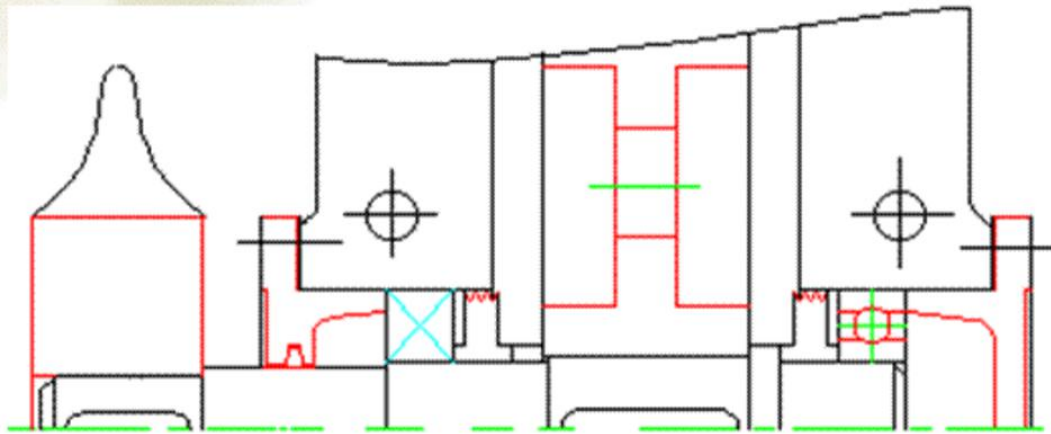
★ A single keyway 单键槽: d_{\min} 增大 3%;

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★ Two keyways 双键槽: d_{\min} 增大 7%

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二、 Influencing factors on structural design 结构设计的影响因素



❖ 1、 Structural composition 结构组成
Shaft end 轴头、
Shaft journals 轴颈、
Shaft body 轴身

❖ 2、 Structural design 结构设计：
1) Locating of the components on
the shafts 轴上零件定位：

① Axial locating 轴向定位



为了防止零件在载荷作用下轴向移动,保证零件的准确工作位置,零件在轴上必须作轴向定位与固定。

Shaft shoulder or ring

轴上零件轴向定位与固定的方法通常有:轴肩 (或轴环)、挡圈 (弹性挡圈、螺钉锁紧挡圈、轴端挡圈)、圆螺母、套筒、圆锥形轴头等。

Snap ring, end washer, locknut, barrel, tapered shaft end



Decrease stress concentration 减小应力集中

The main measures of decreasing stress concentration:

改进轴的结构,减小应力集中的主要措施有:

Providing load relief grooves on shaft or hubs

in

el

us of

1. 在轴上或轮毂上开减载槽;
2. 内圆角、凹切圆角、肩环或柔性轮毂;
3. 避免在轴上打印、键槽圆角过小等;
4. 盘铣刀开键槽;
5. 轴上横孔孔端倒角。

② Circumferential locating 周向定位

为了满足机器传递运动和扭矩的要求,轴上零件还必须作可靠的周向固定,常用的周向固定方法有:

1. 键联接 Key joints The frequent methods:
2. 花键联接 Spline joints
3. 成型联接 Shaped joints
4. 弹性环联接 Elastic ring joints
5. 销联接 Pin joints
6. 过盈联接等 Interference fit joints

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3 Improvement of the force conditions 改善受力状态

为了提高轴的承载能力和减小轴的尺寸,可从如下诸方面采取措施。

Make shaft only bear bending moment and do not bear torque

使轴只受弯矩、不受转矩

Make reasonable arrangement of the components on the shaft

零件在轴上的布置

Assure reasonable structures of the components on the shaft

轴上零件结构的合理性

Make reasonable election of the supporting points of the shaft

轴的支点位置

Make forces on the shaft balance or counteract each other partially

力平衡或局部相互抵消

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§ 10-3 Strength check for the shafts

轴的强

一度校核

一、Allowable bending stress method (common shafts)
许用弯曲应力法（一般用途轴）

二、Safety factor method (heavy-load shaft:
stress concentration is taken into
account) 安全系数法（重要轴：考虑到应力集中）



一、Allowable bending stress method (for common shafts)

许用弯曲应力法（一般用途轴）

❖ 1、Bending and torsion moment

diagrams are constructed.

And dangerous cross sections are
determined. 绘制弯矩扭矩（MT）图，确定危险截
面



2、 Strength check 强度校核

Usually, shafts bear bending moment and torque. Hence, according to the maximum shear theory of failure, we can obtain strength calculation formula of the shafts.

轴一般是弯矩和扭矩联合作用，根据第3强度理论（ τ_{\max} 理论）进行计算：

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❖1) Equivalent stress 当量应力

$$\sigma_v = \sqrt{\sigma_b^2 + 4\tau^2} = \sqrt{\left(\frac{M}{W}\right)^2 + 4\left(\frac{T}{2W}\right)^2} = \frac{1}{W} \sqrt{M^2 + T^2} \leq [\sigma_{-1}]$$
$$\begin{cases} \sigma_b = M/W \\ \tau = T/W_P = \frac{T}{2W} \end{cases} \quad W = \frac{\pi d^3}{32} \approx 0.1d^3$$

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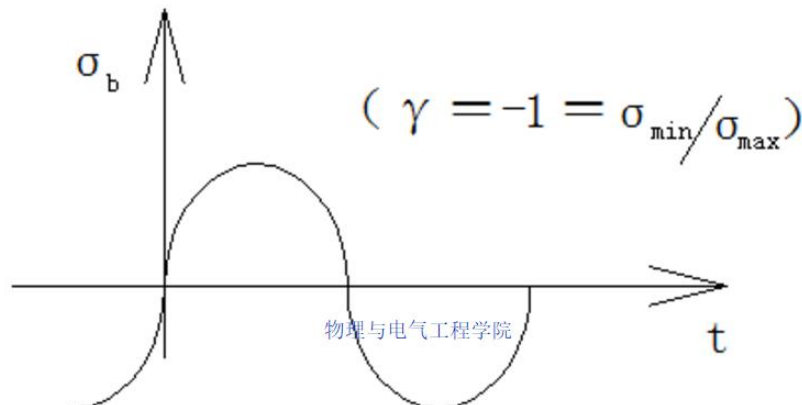
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2) Rotating shaft results in stress change

轴转动，导致应力变化

Bending stress change in common transmission shafts:

一般转轴弯曲应力 σ_b 的变化规律：



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Torsion stress changes in common transmission shafts:

一般转轴扭剪应力 τ 的变化规律：

Stress correcting factor

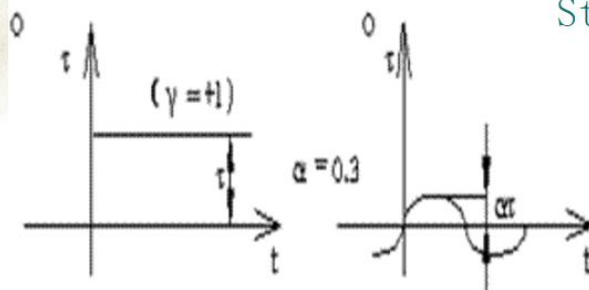


图 a

α 为应力校正系数

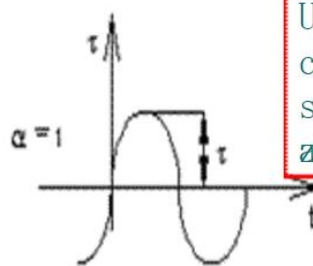
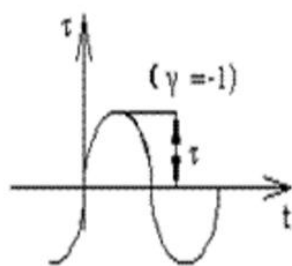
α 的意义：

把其它循环变化的
剪应力转化为
对称循环剪应力

Change torsion stress in
other alternating cycles
into torsion stress in symmetrical
alternating cycles

图 b

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Under common condition, the torsion stress is considered zero-plus cycle

$$\alpha = \frac{[\sigma_+]}{[\sigma_-]}$$

Under Ideal condition, the torque is constant and equivalent to static load

图a: 理想情况, τ 不变, 相当于静载荷

图b: 一般转轴, 认为是脉动循环变化

图c: 经常正反转的轴, 认为是对称循环变化

Strength condition

强度条件:

$$\sigma_v = \frac{1}{0.1d^3} \sqrt{M^2 + (\alpha T)^2} \leq [\sigma_+]$$

Under frequent back running condition, the torsion stress is considered as symmetrical cycle

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二、Safety factor method (heavy-load shaft, stress concentration is taken into account)

安全系数法 (重要轴: 考虑到应力集中)

❖ Strength condition 强度条件:

$$S \geq [S]$$

Safety factor

S — 综合安全系数

$[S]$ — 许用安全系数

Endurance safety factor

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❖ 1、[S] ——P118

With accurate calculations and homogeneous materials: a lower endurance safety factor can be chosen.

$$[S] \geq 1.3 \sim 1.5$$

With common calculations and common materials: the endurance safety factor can be chosen as follows,

With rough calculations and heterogeneous materials: the endurance safety factor can be chosen as follows

$$[S] \geq 1.8 \sim 2.5$$



❖ 材质均匀、载荷与应力计算精确时，

[S] 材质不够均匀、载荷与应力计算不够精确时，

[S] 材质均匀性很低、载荷与应力计算精度很低时，

$$[S] \geq 1.8 \sim 2.5$$

❖2、S

$$S = \frac{S_\sigma S_\tau}{\sqrt{S_\sigma^2 + S_\tau^2}}$$

$$S_\sigma = \frac{K_{N\sigma} - 1}{(K_\sigma)_D \times \sigma_a + \psi_\sigma \sigma_m}$$

$$S_\tau = \frac{K_{N\tau} - 1}{(K_\tau)_D \times \tau_a + \psi_\tau \tau_m}$$



(1) $\sigma_a, \sigma_m, \tau_a, \tau_m$ Find according to forces of the shaft 根据受力情况求出

(2) $(K_\sigma)_D, (K_\tau)_D$ - Synthetic stress concentration factor 综合应力集中系数

$$\left\{ \begin{array}{l} (K_\sigma)_D = \frac{K_\sigma}{\beta \epsilon_\sigma} \quad \text{① Interference fit 过盈配合 } (K_\sigma)_D, (K_\tau)_D \quad \text{P126附表3} \\ (K_\tau)_D = \frac{K_\tau}{\beta \epsilon_\tau} \quad \text{② Fillet 过渡圆角} \left\{ \begin{array}{l} K_\sigma, K_\tau - \text{Effective stress concentration factor} \\ \text{有效应力集中系数} \quad \text{P129附表1} \\ \epsilon_\sigma, \epsilon_\tau - \text{Scale factor 尺寸系数} \quad \text{P130附表6} \\ \beta - \text{hardening factor 表面状态系数} \quad \text{P129附表4} \end{array} \right. \\ \text{③ Keyway 键槽: } K_\sigma, K_\tau - \text{P129附表2; } \epsilon_\sigma, \epsilon_\tau, \beta \text{ 同上} \end{array} \right.$$

(3) σ_{-1} , τ_{-1} , Ψ_σ , Ψ_τ

σ_{-1} , τ_{-1} — Endurance limits for bending

and torsion in symmetrical alternating cycles

对称循环载荷下，无应力集中试样疲劳极限

σ_0 , τ_0 — Endurance limits for bending

and torsion in zero - plus alternating cycles

脉动循环载荷下，无应力集中试样疲劳极限

Ψ_σ , Ψ_τ — Factors indicating the sensitivity of the material
to asymmetry of the stress cycle

平均应力 σ_m , τ_m 折算为应力幅 σ_a , τ_a 的等效系数
(材料对对称循环载荷的敏感系数)

$$\Psi_\sigma = \frac{2\sigma_{-1} - \sigma_0}{\sigma_0}$$

$$\Psi_\tau = \frac{2\tau_{-1} - \tau_0}{\tau_0}$$

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(4) K_N 寿命系数 一般取无限寿命 $K_N = 1$

Endurance factor can be taken as 1 according to infinite endurance

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3、 Under heavy-load or impact-load:
static strength check (for plastic flow of
materials)

重载 或冲击载荷较大时：静强度校核（
防止塑性变形）

$$S \geq [S]$$

S — Safety factor

综合安全系数

$[S]$ — Allowable safety factor

许用安全系数

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$$S = \frac{S_\sigma S_\tau}{\sqrt{S_\sigma^2 + S_\tau^2}}$$

$$S_\sigma = \frac{\sigma_s}{\sigma_{\max}}$$

$$S_\tau = \frac{\tau_s}{\tau_{\max}}$$

$[S]$ — P 118 表 8 — 4



4、 Dangerous cross section 危险剖面



- size of
- ① According to the stress
(According to the moment diagrams and the shaft cross sections)
应力大小 (弯矩M大小、扭矩T有无、轴径d大小)
 - ② According to the stress concentration
(interference fit, keyway and fillet)
有无应力集中 (过盈配合、键槽、过渡圆角)

第4节 轴的振动计算和临界转速



工作转速 n 低于第一阶临界转速 n_{cr1} 的轴，称为刚性轴；
超过第一阶临界转速的轴，称为挠性轴。



对于刚性轴，通常使 $n < (0.75 \sim 0.8) n_{cr1}$ ；对于挠性轴，使 $4n_{cr1} \leq n \leq 0.72n_{cr2}$ ； n_{cr1} 和 n_{cr2} 分别为轴的第一阶和第二阶临界转速。

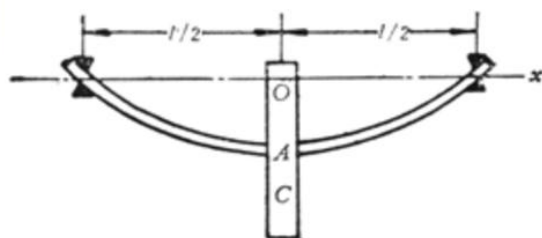


❖ e 是由于材料的不均匀性和制造上的不精确性等原因而存在的小偏心距。

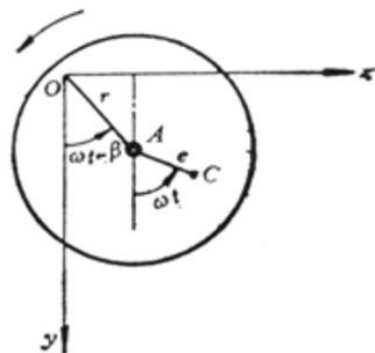
❖ 当轴以角速度 ω 转动时，就产生一个随着圆盘转动的离心力

:

$$Q_0 = me \omega^2$$



(a)



(b)

装有单圆盘的双铰支轴

❖这个离心力在y轴和z轴上的分力是：

$$Q_y = Q_0 \cos \omega t = m\omega^2 \cos \omega t$$

$$Q_z = Q_0 \sin \omega t = m\omega^2 \sin \omega t$$

这两个周期性变化的分力 Q_y 和 Q_z 分别在y轴和z轴方向上同时掀起强迫振动。

❖ 激振力产生的静位移为：

$$x_s = \frac{Q_0}{k} = \frac{\frac{Q_0}{m}}{\frac{k}{m}} = \frac{e \omega^2}{p^2}$$

❖ 根据有关强迫振动稳态解的公式就可以直接写出圆盘中心点A在y轴和z轴方向上的运动方程为：

见下页

$$y = \frac{e\left(\frac{\omega}{p}\right)^2}{\sqrt{\left[\left(1 - \frac{\omega^2}{p^2}\right)^2 + \left(2c\frac{\omega}{p}\right)^2\right]}} \cos\left\{\omega t - \tan^{-1}\left[\frac{2c\frac{\omega}{p}}{1 - \left(\frac{\omega}{p}\right)^2}\right]\right\}$$

$$z = \frac{e\left(\frac{\omega}{p}\right)^2}{\sqrt{\left[\left(1 - \frac{\omega^2}{p^2}\right)^2 + \left(2c\frac{\omega}{p}\right)^2\right]}} \sin\left\{\omega t - \tan^{-1}\left[\frac{2c\frac{\omega}{p}}{1 - \left(\frac{\omega}{p}\right)^2}\right]\right\}$$

❖ 式中的 p 是轴-盘系统作横向自由振动(弯曲振动)的圆频率, 由机械振动理论知:

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\delta_s}} = \sqrt{\frac{48EI}{ml^3}}$$

- ❖ 圆盘中心点 A 的运动轨迹是个圆，圆心是点 I ，半径是：

$$r = \frac{e \left(\frac{\omega}{p} \right)^2}{\sqrt{\left[\left(1 - \frac{\omega^2}{p^2} \right)^2 + \left(2c \frac{\omega}{p} \right)^2 \right]}}$$

- ❖ 当忽略阻尼 ($c=0$) 时，则轴的挠度是：

$$r = \frac{e \left(\frac{\omega}{p} \right)^2}{1 - \left(\frac{\omega}{p} \right)^2}$$

- ❖ 当 ω 在数值上愈接近 p 时，轴
- ❖ 的挠度 r 就愈要增大；当 $\omega = p$ 时， $r \rightarrow \infty$ ，这时出现共振，
- ❖ 轴便发生大幅度的横向振动。

- ❖ 因此也就证明了临界转速（ ω_{cr} ）就是与轴的弯曲自振圆频率 p 相等的角速度，即：

$$\omega_{cr} = p = \sqrt{\frac{g}{\delta_s}} = \sqrt{\frac{48EI}{ml^3}}$$

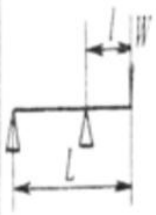

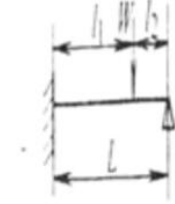


$$n_{cr} = \frac{30}{\pi} \omega_{cr} \text{ (r/min)} \quad (7-21)$$

❖式（7-21）即为圆盘相
对于两铰支点对称分布
时的临界转速计算公式。



❖同理，可以得到不计轴的质量
时单圆盘相对于其它支承方式
及非对称布置的临界转速如表
所示。

两支承单盘轴的临界转速

支座形式					
临界转速 $n_{cr}/(r \cdot \min^{-1})$	$\frac{30}{\pi} \sqrt{\frac{3EI}{ml^2 L}}$	$\frac{30}{\pi} \sqrt{\frac{3EIL}{ml_1^2 l_2^2}}$	$\frac{30}{\pi} \sqrt{\frac{12EIL^3}{ml_1^2 l_2^2 (3L + l_2)}}$	$\frac{30}{\pi} \sqrt{\frac{3EI}{mL^3}}$	$\frac{30}{\pi} \sqrt{\frac{3EIL^3}{ml_1^2 l_2^2}}$

E —材料弹性模量, Pa; I —轴截面惯性矩, m^4 ; L —跨距, m; $m = \frac{W}{g}$ —质量, kg